Optimal Control of Robotic Grasping

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Abstract
A control technique for the control of contact forces between the links of a multiple-chain robot system (such as a robot hand) and an object is presented. The goal of the control method is to optimise contact forces so as to minimise a cost function, corresponding to minimisation of a weighted sum of factors such as energy consumption and sensitivity to force disturbances. A globally stable algorithm is provided, that asymptotically converges to the optimum.

1 Introduction

The force and moment balance equations for an object subject to an external force $\mathbf{f}$ and moment $\mathbf{m}$, while grasped by a robotic mechanism by means of n contact forces $\mathbf{p}_i$, can be written in matrix notation as

$$\mathbf{w} = \mathbf{G}\mathbf{t}, \qquad (1)$$

where $\mathbf{w} = (\mathbf{f}^T, \mathbf{m}^T)^T$, $\mathbf{t} = (\mathbf{p}_1^T, \dots, \mathbf{p}_n^T)^T$, and

$$\mathbf{G} = \begin{pmatrix} \mathbf{I}_3 & \cdots & \mathbf{I}_3 \\ \mathbf{c}_1\times & \cdots & \mathbf{c}_n\times \end{pmatrix}.$$

The relationship between contact forces and the torques at the m joints can be written as

$$\tau = \mathbf{J}^T \mathbf{t},$$

where $\mathbf{J}$ is the equivalent of the jacobian matrix for conventional manipulators. We assume that, in a neighborhood of the equilibrium configuration under investigation, $\mathbf{G}$ and $\mathbf{J}$ are full row rank. A general solution of (1) can be written as

$$\mathbf{t} = \mathbf{G}^R\mathbf{w} + \mathbf{E}\mathbf{y}, \qquad (2)$$

i.e., the sum of a particular solution of (1) ($\mathbf{G}^R$ standing for a generic right-inverse of $\mathbf{G}$), and a homogeneous solution ($\mathbf{E}$ being a basis matrix of the nullspace of $\mathbf{G}$). The coefficient vector $\mathbf{y} \in \Re^h$ parametrises the homogeneous solution. "Internal" contact forces $\mathbf{t} = \mathbf{E}\mathbf{y}$ have no direct effect on the external force $\mathbf{w}$. However, they play an important role in the robustness of the equilibrium w.r.t. slippage induced by external disturbances. Coulomb's law of friction can be written for each contact point as:

$$\mathbf{p}_i^T \mathbf{n}_i \geq \alpha_i \|\mathbf{p}_i\|, \qquad (3)$$

where $\mathbf{n}_i$ is the unit vector normal to the object surface at $\mathbf{c}_i$. Internal forces also contribute to the local contact force intensity, thus influencing the danger of damage on fragile objects, and the energy spent for maintaining the desired equilibrium. This suggests to keep contact forces below a suitable threshold:

$$\|\mathbf{p}_i\| \leq f_{i,max} > 0. \qquad (4)$$

More constraints on contact forces may be added depending on the particular task. Their treatment is analogous, and omitted here for brevity. The problem of optimally choosing internal forces has been extensively studied, mostly as a constrained programming problem in a linearised (e.g., [1]) or non-linear setting ([2]). In our approach a more efficient, globally asymptotically convergent algorithm is obtained which realises the goal of keeping forces as far as possible from violation of constraints (3) and (4).

2 Cost Function

Note that constraints (3) and (4) on the i-th contact force can be written in the form

$$\sigma_{i,j}(\mathbf{y}) = \alpha_{i,j}\,\|\mathbf{p}_i\| + \beta_{i,j}\,\mathbf{p}_i^T\mathbf{n}_i + \gamma_{i,j} \leq 0, \qquad (5)$$

where $\alpha_{i,1} = \alpha_i$, $\beta_{i,1} = -1$, and $\gamma_{i,1} = 0$ for friction constraints; $\alpha_{i,2} = 1$, $\beta_{i,2} = 0$, and $\gamma_{i,2} = -f_{i,max}$ for maximum force constraints.

Let $\Omega^{\kappa}_{i,j} \subset \Re^h$ indicate the set of grasp variables that, in the presence of a given load $\mathbf{w}$, satisfy constraints (5) of corresponding indices with a (small, positive) margin κ, $\Omega^{\kappa}_{i,j} := \{\mathbf{y} \mid {}^j\sigma_i(\mathbf{y}) < -\kappa\}$. For the i-th contact and the j-th constraint, consider the cost function

$$V_{i,j}(\mathbf{y}) = \begin{cases} (2\,\sigma_{i,j}^2)^{-1} & \mathbf{y} \in \Omega^{\kappa}_{i,j} \\ a\,\sigma_{i,j}^2 + b\,\sigma_{i,j} + c & \mathbf{y} \notin \Omega^{\kappa}_{i,j} \end{cases}, \qquad (6)$$

An overall cost function is defined as the sum of such terms:

$$V(\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{2} w_{i,j} V_{i,j}(\mathbf{y}), \qquad (7)$$

where $w_{i,j} > 0$ are suitable weights. By partitioning (2) as

$$\mathbf{t} = \begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_n \end{pmatrix} = \mathbf{G}^R\mathbf{w} + \mathbf{E}\mathbf{y} = \begin{pmatrix} \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_n \end{pmatrix}\mathbf{w} + \begin{pmatrix} \mathbf{M}_1 \\ \vdots \\ \mathbf{M}_n \end{pmatrix}\mathbf{y},$$

we have

$$\mathbf{p}_i(\mathbf{y}) = \mathbf{P}_i\,\mathbf{w} + \mathbf{M}_i\,\mathbf{y}. \qquad (8)$$

The gradient of the cost function with respect to $\mathbf{y}$ is the weighted summation over i and j of the terms

$$\frac{\partial V_{i,j}}{\partial \mathbf{y}} = \begin{cases} -\frac{1}{\sigma_{i,j}^3} \frac{\partial \sigma_{i,j}}{\partial \mathbf{y}}, & \mathbf{y} \in \Omega_{i,j}^{\kappa} \\ (2a\, \sigma_{i,j} + b) \frac{\partial \sigma_{i,j}}{\partial \mathbf{y}} & \mathbf{y} \notin \Omega_{i,j}^{\kappa} \end{cases} \quad (9)$$

$$\frac{\partial \sigma_{i,j}}{\partial \mathbf{y}} = \alpha_{i,j}\, \mathbf{M}_i^T \vec{\mathbf{p}}_i + \beta_{i,j}\, \mathbf{M}_i^T \mathbf{n}_i, \quad (10)$$

and $\vec{\mathbf{p}} = \mathbf{p}/||\mathbf{p}||$.The cost function hessian is the weighted summation of the terms

$$\frac{\partial^2 V_{i,j}}{\partial \mathbf{y}^2} = \begin{cases} -\frac{1}{\sigma_{i,j}^3} \frac{\partial^2 \sigma_{i,j}}{\partial \mathbf{y}^2} + \frac{3}{\sigma_{i,j}^4} \frac{\partial \sigma_{i,j}}{\partial \mathbf{y}} \frac{\partial \sigma_{i,j}^T}{\partial \mathbf{y}}, \\ (2a\sigma_{i,j} + b) \frac{\partial^2 \sigma_{i,j}}{\partial \mathbf{y}^2} + 2a \frac{\partial \sigma_{i,j}}{\partial \mathbf{y}} \frac{\partial \sigma_{i,j}^T}{\partial \mathbf{y}}, \end{cases} \quad (11)$$

$$\frac{\partial^2 \sigma_{i,j}}{\partial \mathbf{y}^2} = \alpha_{i,j} \frac{\mathbf{M}_i^T \left(\mathbf{I} - \vec{\mathbf{p}}_i \vec{\mathbf{p}}_i^T\right) \mathbf{M}_i}{||\mathbf{p}_i||}$$

Imposing twice continuous differentiability of $V_{i,j}$ on the boundaries of $\Omega_{i,j}^{\kappa}$ provides conditions on a, b, and c.

Proposition 1 *The cost function defined in (7) with $a = \frac{3}{2\kappa^4}$, $b = \frac{4}{\kappa^3}$, and $c = \frac{3}{\kappa^2}$, is strictly convex with respect to $\mathbf{y} \in \Re^h$.*

The proof follows from observing that the discontinous terms in (5), (9), and (11) can be regarded as the limits of sequences of functions continuously differentiable over $\Re^h$. Hence, the positive definiteness of the hessian of V is a necessary and sufficient condition for its convexity. Being (11) the summation of matrices which can be trivially shown to be s.p.d., it will suffice to show that the intersection of the nullspaces of each addend is zero. Focusing on terms due to maximum force constraint ($j = 2$), suppose there exists a vector $\mathbf{x} \in \Re^h$ such that, for every i,

$$\mathbf{x}^T \frac{\partial^2 \sigma_{i,j}}{\partial \mathbf{y}^2} \mathbf{x} = \alpha_i \mathbf{x}^T \mathbf{M}_i^T \left(\mathbf{I} - \vec{\mathbf{p}}_i^T \vec{\mathbf{p}}_i\right) \mathbf{M}_i \mathbf{x} = 0;$$

$$\mathbf{x}^T \frac{\partial \sigma_{i,j}}{\partial \mathbf{y}} \frac{\partial \sigma_{i,j}^T}{\partial \mathbf{y}} \mathbf{x} = \alpha_i \mathbf{x}^T \mathbf{M}_i^T \vec{\mathbf{p}}_i \vec{\mathbf{p}}_i^T \mathbf{M}_i \mathbf{x} = 0.$$

These conditions imply that $\mathbf{M}_i\mathbf{x}$ should be parallel and normal to $\vec{\mathbf{p}}_i$, respectively. The only solution is for $\mathbf{M}_i\mathbf{x} = 0$. Since this must hold for every i, by juxtaposing all such relationship we have the condition $\mathbf{E}\mathbf{x} = 0$. Being the columns of $\mathbf{E}$ independent (they form a basis of the subspace of homogeneous solutions), it follows $\mathbf{x}^T \frac{\partial^2 V}{\partial \mathbf{y}^2} \mathbf{x} > 0, \forall \mathbf{x} \neq 0$. □

3 Control algorithm

The aim of this section is to design a suitable law for controlling contact forces in the grasp of an object, which is subject to an external force $\mathbf{w}$. Such forces are assumed bounded and resistible (i.e., there exists at least one possible solution to the grasp equation (1) with constraints (4), and (3)). Moreover, we assume $\mathbf{w}$ to vary slowly, so that $\dot{\mathbf{w}} \approx 0$.

Proposition 2 *Assume that an object in a stable grasp is subject to forces and torques $\mathbf{w}$ that are resistible. Then, for $\zeta > 0$ and any initial condition $\mathbf{y}(0) = \mathbf{y}_o$, the joint torque control law*

$$\tau(t) = -\,\mathbf{J}^T\, \mathbf{G}^R\, \mathbf{w} \,+\, \mathbf{J}^T\, \mathbf{E}\, \mathbf{y}(t); \quad (12)$$

with the update law

$$\dot{\mathbf{y}}(t) = -\zeta \frac{\partial^2 V}{\partial \mathbf{y}^2}^{-1} \frac{\partial V}{\partial \mathbf{y}}, \quad (13)$$

ensures that the object equilibrium is maintained, while asymptotically converging to the optimal (in the sense of minimizing the cost (7)) set of contact forces.

Clearly, the first term on the right-hand side of (12) ensures that equilibrium is maintained. Since the cost function has been shown to be strictly convex, the dynamics of the optimisation parameter vector $\mathbf{y}$ defined in (13) have an unique equilibrium point in $\hat{\mathbf{y}}$, where the cost gradient vanishes. To show the global asymptotic stability of $\hat{\mathbf{y}}$, introduce $\mathbf{e}(t) = \mathbf{y}(t) - \hat{\mathbf{y}}$, and consider the p.d., radially unbounded Lyapunov candidate $V(\mathbf{e})$ obtained from (7):

$$\dot{V} = \frac{\partial V}{\partial \mathbf{e}}^T \dot{\mathbf{e}} = -\zeta \frac{\partial V}{\partial \mathbf{y}}^T \frac{\partial^2 V}{\partial \mathbf{y}^2}^{-1} \frac{\partial V}{\partial \mathbf{y}} \quad (14)$$

which is clearly negative definite.□

Note that, should $\mathbf{J}$ be less than full row rank, an arbitrary contact force $\mathbf{t}$ may not be realised by controlling joint torques. In this case, which is of relevance to robot systems using all their parts to manipulate objects (e.g., in "power grasping" and in "whole-arm manipulation"), the optimal contact force must be chosen in the range space of a suitably modified $\mathbf{E}$ matrix.

In (12), it is assumed that $\mathbf{w}$ is known. This is true in the case that $\mathbf{w}$ is a pre-planned force-trajectory to be realised by the object (used as a tool) upon the environment. Thus, a feed-forward control scheme is realised. In alternative, $\mathbf{w}$ can be measured by force sensors. Note however that, once the static gain of the joint position controller loops is set, the particular solution of (1) corresponding to the physical distribution of contact forces to balance $\mathbf{w}$ establishes joint torques upon which the optimising term can be superimposed.

Although the control algorithm has been discussed in the continuous time domain, it is straightforward to derive its discrete time analog. In this case, however, the global asymptotic convergence of the algorithm can be proven only for values of ζ smaller than a limit value. Such limitations on ζ will only allow the convergence to a finite neighborhood of the optimal grasp.

The discussed method has been successfully simulated in various conditions for different robotic mechanisms. Preliminary experimental results have also been obtained, showing good applicative potentiality of the method.

References

[1] Orin, D.E., and Oh, S.Y.: "Control of Force Distribution in Robotic Mechanisms Containing Closed Kinematic Chains", J. Dyn. Sist. Meas. Contr., vol. 102, 1981.

[2] Nakamura, Y., Nagai, K., and Yoshikawa, T.: "Dynamics and Stability in Coordination of Multiple Robotic Systems", International Journal of Robotic Research, vol.8, no.2, April 1989.