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A Diagonal Dominance Condition for Robust Decentralized Variable Structure Control

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Abstract

In this paper, the robust tracking control of MIMO systems by means of simple decentralized variable-structure controllers is considered. The model of the system that is assumed to be available is a linear i/o map (e.g., a transfer function matrix) affected by bounded uncertainties and subject to bounded input disturbances. A sufficient condition is derived for there to exist the possibility of exact tracking by decentralized sliding-mode control. The condition can be seen as a time-domain counterpart to Rosenbrock's well-known diagonal dominance criterion. The structure of a simple controller implementing the technique is presented along with simulation results.

1 Introduction

Decentralized controllers attracted much attention in the last two decades, especially in relation with large-scale systems such as in electric power systems, socioeconomic systems, chemical processes, etc., where reducing the controller complexity is a major concern of the control system designer.

The literature on decentralized control can be grossly divided in two main branches, dealing with methods in the frequency and in the time domains, respectively. Frequency domain methods have attracted the interest of researchers due to the fact that available models of large-scale systems are often of the input-output type, mostly in the form of approximated transfer function matrices. Rosenbrock's DNA and INA techniques [Rosenbrock, 1974] for the design of decentralized linear controllers for linear multivariable systems have proved to be among the most effective and practical tools for approaching large-scale systems that exhibit weak coupling among SISO subsystems.

Rosenbrock's necessary dominance conditions are generally recognized to be rather difficult to achieve in real applications. This led a number of researchers to investigate techniques for relaxing the condition and generalize the method. Generalized and block-diagonal dominance conditions have been proposed to this effect (see e.g. [Ohta, Siliak, and Matsumoto, 1986]).

Time-domain methods can in turn be distinguished between linear decentralized controllers for robust tracking (see e.g. [Siliak, 1978]; [Chen *et al.*, 1991]), and variable-structure controllers ([Utkin, 1977]). Lefebvre *et al.* [1982], Khurana *et al.* [1986] studied decentralized VS stabilizing controllers, while the tracking controller problem for a class of interconnected multivariable systems has been given a solution by Matthews and DeCarlo [1988].

In this paper, the connections between frequency-domain and VSC techniques for decentralized control of general multivariable systems are investigated. In order to retain the practice-oriented flavour of frequency-domain methods, the assumed model of the plant is an input/output relationship represented by a transfer function matrix $\mathbf{G}(s)$. As it is customary in such approach, nonlinearities in the system are dealt with as uncertainties in the model, and input disturbances of bounded norm are also included. As an interesting result, this paper discusses a sufficient condition for a system to allow the existence of a VS decentralized controller guaranteeing zero tracking error in finite time. The condition is shown to be closely related (in the time-domain) to Rosenbrock's frequency-domain dominance condition.

2 Dominance condition

Let us consider the $N \times N$ transfer function matrix $\mathbf{G}(s)$ of a MIMO system with inputs u_i and outputs y_i, which is decomposed as

$$\mathbf{G}(s) = \mathbf{G}_D(s) + \mathbf{G}_C(s)$$

with

$$\mathbf{G}_D(s) = \text{diag}\,\{G_i(s)\}, \qquad \mathbf{G}_C(s) = \{G_{ij}(s)\}$$

where $\mathbf{G}_C(s)$ reflects the uncertainty in the diagonal subsystems $G_i(s)$ and the subsystem interconnections. Consider N [respectively N^2] SISO systems $\Sigma_i = (\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i, \mathbf{d}_i)$ [$\Sigma_{ij} = (\mathbf{A}_{ij}, \mathbf{b}_{ij}, \mathbf{c}_{ij}, \mathbf{d}_{ij})$], each providing a minimal realization of order n_i [n_{ij}] and relative order r_i [r_{ij}] of element $G_i(s)$ [$G_{ij}(s)$]. The i-th output y_i of the plant can be expressed in terms of the disturbed output of the diagonal system Σ_i

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{A}_i\mathbf{x}_i + \mathbf{b}_i(u_i + \nu_i), & \mathbf{x}_i(0) \stackrel{def}{=} \mathbf{x}_i^o \\ y_i = \mathbf{c}_i\mathbf{x}_i + \mathbf{d}_i(u_i + \nu_i) + \varphi_i \end{cases}$$

where $\nu_i(t)$ is a bounded input disturbance, $\|\nu_i(t)\|_\infty < N_i$, representing noise on the actuators

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and possibly nonlinear dynamic terms assolving the ***matching condition*** (cf. eg. [Walkott and Zak, 1988]), and the output disturbance is given by

$$\varphi_i(t) \stackrel{def}{=} \sum_{j=1,N} \varphi_{ij}(t),$$
$$\left\{ \begin{array}{rcl} \dot{x}_{ij} & = & A_{ij}x_{ij} + b_{ij}(u_j + \nu_j), \ x_{ij}(0) = x_{ij}^o \\ \varphi_{ij} & = & c_{ij}x_{ij} + d_{ij}(u_j + \nu_j) \end{array} \right.$$

Consider further N systems $\mathcal{S}_{z_i}$ of order n_i,

$$\left\{ \begin{array}{rcl} \dot{z}_i & = & \bar{A}_i z_i + \bar{b}_i \psi_i, \ z_i(0) = 0 \\ w_i & = & \bar{c}_i z_i + \bar{d}_i \psi_i \end{array} \right. \quad (1)$$

We are interested in conditions for $\psi_i(t)$ under which outputs $w_i(t)$ match $y_i(t)$. To this regard, simple results can be obtained by choosing the pairs $(\bar{A}_i, \bar{b}_i, \bar{c}_i, \bar{d}_i)$ and (A_i, b_i, c_i, d_i) to be algebraically equivalent:

Lemma 1 *Assume*

H1	$\inf_{\text{Re } s \geq 0} \lvert G_i(s)\rvert \ > 0$	$\forall i$
H2	$\sup_{\text{Re } s \geq 0} \lvert G_{ij}(s)\rvert \ \leq M \in \mathbb{R}_+$	$\forall i, j$
H3	$\min_j\{r_{ij} - r_i\} \ \geq 0,$	$\forall i;$

U.t.c., there exist tempered generalized functions $\psi_i(t) = u_i(t) + \nu_i(t) + \zeta_i(t)$ such that $w_i(t) = y_i(t), \forall t > 0$. Distributions $\zeta_i(t)$ may contain delta functions and derivatives of delta functions up to the $(r_i - 1)$-th order. Furthermore, if the plant is initially relaxed and $\nu_i \in L^p$, then $\zeta_i \in L^p$.

Proof. Let $g_{ij}(t)$ denote the impulse response of $G_{ij}(s)$, and $\mathcal{A}$ denote the algebra of stable impulse responses (see e.g. [Vidyasagar, 1978]). By equating $y_i(t)$ and $w_i(t)$,

$$g_i * \zeta_i = c_i \exp(A_i t) x_i^o +$$
$$+ \sum_{j=1,N} \left(g_{ij} * (u_j + \nu_j) + c_{ij} \exp(A_{ij} t) x_{ij}^o \right)$$

By hypothesis H2, $g_{ij}(t) \in \mathcal{A}$. Moreover, let $\hat{g}_i(t)$ be defined such that $\hat{g}_i(t) * g_i(t) = \delta(t)$, the unit element in $\mathcal{A}$. Hence, by H1 and H3, $\hat{g}_i * g_{ij} \in \mathcal{A}$, and

$$\zeta_i(t) = \sum_{j=1,N} (\hat{g}_i * g_{ij}) * (u_j(t) + \nu_j(t)) +$$
$$+ \hat{g}_i * \left(\sum_{j=1,N} c_{ij} \exp(A_{ij} t) x_{ij}^o + c_i \exp(A_i t) x_i^o \right)$$

For arbitrary initial conditions, the second term on the left-hand side contains singular functions δ^d with $d \leq (r_i - 1)$. For relaxed initial conditions, it holds

$$\|\zeta_i(t)\|_p \leq \sum_{j=1,N} \|\hat{g}_i * g_{ij}\|_{\mathcal{A}} (\|u_j\|_p + \|\nu_j\|_p) =$$
$$\sum_{j=1,N} \left(\|\hat{g}_i * g_{ij}\|_1 + G_i^{-1}(s) G_{ij}(s) \big|_{s \to \infty} \right) (\|u_j\|_p + \|\nu_j\|_p)$$

□.

Consider now the synthesis of a variable structure control $u_i(t)$ for $\mathcal{S}_{z_i}$, that is assumed without loss of generality to be in canonical controller form,

$$\left\{ \begin{array}{rcl} \dot{z}_i & = & \bar{A}_i z_i + \bar{b}_i(\nu_i + \zeta_i) + \bar{b}_i(u_i), \ z_i(0) = 0 \\ w_i & = & \bar{c}_i z_i + \bar{d}_i(\nu_i + \zeta_i) + \bar{d}_i(u_i) \end{array} \right. .$$

To regulate the output to zero with prescribed dynamics a sliding manifold is designed as

$$\mathcal{Z}_i : \ s_i(t) \stackrel{def}{=} \eta_i z_i(t) = 0 \quad (2)$$

where $\eta_i \in \mathbb{R}^{n_i}$. Since (2) can be scaled without affecting the dynamics of the sliding regime, we choose $\eta_i = [\ \eta_i' \ \ 1 \ ]$ so that $\eta_i \bar{b}_i = 1$. Pole assignment and optimal LQ techniques are usually employed for choosing η_i', as described e.g. by Dorling and Zinober [1986]. The equivalent control [Utkin, 1977] is

$$u_{eq_i}(t) = -\eta_i \bar{A}_i z_i - \nu_i - \zeta_i \stackrel{def}{=} -u_i' - \nu_i - \zeta_i$$

Convergence towards $\mathcal{Z}_i$ after a certain time t_{si} is guaranteed by the condition

$$s_i(t)\dot{s}_i(t) < 0, \ \ \forall t \geq t_{si}. \quad (3)$$

By applying the variable structure control

$$u_i(t) = -k_i \ \text{sign}\ (\eta_i z_i(t)), \quad (4)$$

condition (3) is met if

$$k_i > \|u_{eq_i}(t)\|_\infty^{t_{si}} \stackrel{def}{=} \sup_{t > t_{si}} |u_{eq_i}(t)|.$$

In particular, by choosing

$$k_i = \|u_i'(t)\|_\infty^{t_{si}} + \|\nu_i(t)\|_\infty^{t_{si}} + \|\zeta_i(t)\|_\infty^{t_{si}} + \epsilon_i \quad (5)$$

with $\epsilon_i > 0$, the sliding manifold will be reached in a finite time $t_{si} + t_{ri}$, with $t_{ri} \leq |s_i(t_{si})|/\epsilon_i$.

2.1 Existence conditions

We now indagate conditions under which trajectories of system $\mathcal{S}_{z_i}$ will remain on the sliding manifold $\mathcal{Z}_i$, given that they reach $\mathcal{Z}_i$ at time t_{ri}. In terms of the transformed state variables $\hat{z}$

$$\left[\begin{array}{c} \hat{z}_1 \\ \hat{z}_2 \end{array} \right] = T_i z_i; \qquad T_i \stackrel{def}{=} \left[\begin{array}{cc} I_{n_i - 1} & O_{(n_i-1)\times 1} \\ \eta_i' & 1 \end{array} \right]$$

the sliding equation is $\hat{z}_2 = 0$ and the $(n_i - 1) - th$ order sliding dynamics for $t > t_{ri}$ are

$$\dot{\hat{z}}_1 = {}^c\bar{A}_i \hat{z}_1, \ \text{where } {}^c\bar{A}_i = \left[\begin{array}{cc} O_{(n_i-2)\times 1} & I_{n_i-2} \\ & \eta_i' \end{array} \right].$$

The linear component of the equivalent control $u_i'(t)$ during sliding is given by

$$u_i'(t) = \eta_i A_i \left[\begin{array}{c} I \\ -\eta_i' \end{array} \right] \cdot \exp({}^c\bar{A}_i t) \cdot \hat{z}_1(t_{ri}).$$

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Define a ball in the generic vector norm $\|\cdot\|$ as

$$B^z_{\rho_i} \stackrel{def}{=} \{\hat{z} \in \mathbb{R}^{n_i} | \hat{z}_2 = 0; \|\hat{z}_1\| \le \rho_i \in \mathbb{R}_+\} \subset \mathcal{Z}_i.$$

Provided that ${}^c\mathbf{A}_z$ is (chosen to be) Hurwitz, for any $\hat{z}(t_{ri}) \in B^z_{\rho_i}$ the linear part of the equivalent control is in L_p, and, in particular, there exist bounds $U_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\|u'_i\|^{t_{ri}}_\infty \le \left\| \eta_i \mathbf{A}_i \begin{bmatrix} \mathbf{I} \\ -\eta'_i \end{bmatrix} \right\| \| \exp\,({}^c\bar{\mathbf{A}}_i t)\| \rho_i \le U_i(\rho_i)$$

where compatible norms are chosen. Furthermore, in the hypotheses of lemma 1 and for any $\mathbf{x}^\circ_{ij} \in B_{\rho_{ij}}$,

$$B_{\rho_{ij}} \stackrel{def}{=} \{\mathbf{x}_{ij} \in \mathbb{R}^{n_{ij}} | \, \|\mathbf{x}_{ij}\| \le \rho_{ij} \in \mathbb{R}_+\} \subset \mathbb{R}^{n_{ij} \times n_{ij}},$$

and for any $\mathbf{x}^\circ_i \in B_{\rho_i}$,

$$B_{\rho_i} \stackrel{def}{=} \{\mathbf{x}_i \in \mathbb{R}^{n_i} | \, \|\mathbf{x}_i\| \le \rho_i \in \mathbb{R}_+\} \subset \mathbb{R}^{n_i \times n_i},$$

there exist bounds $V_i(\cdot) : \mathbb{R}^n_+ \to \mathbb{R}_+$ such that

$$\left\| \hat{g}_i * \left(\sum_{j=1,N} \mathbf{c}_{ij} \exp\,(\mathbf{A}_{ij} t)\mathbf{x}^\circ_{ij} + \mathbf{c}_i \exp\,(\mathbf{A}_i t)\mathbf{x}^\circ_i \right) \right\|^{t_{ri}}_\infty$$

$$\le \sum_{j=1,N} \|\tilde{\mathbf{c}}_{ij}\| \left\| \exp\,(\tilde{\mathbf{A}}_{ij} t) \right\| \rho_{ij} + \|\tilde{\mathbf{c}}_i\| \left\| \exp\,(\tilde{\mathbf{A}}_i t) \right\| \rho_i$$

$$\le V_i(\rho_{i1}, \ldots, \rho_{iN})$$

where $(\tilde{\mathbf{A}}_{ij}, \mathbf{x}^\circ_{ij}, \tilde{\mathbf{c}}_{ij})$ [resp. $(\tilde{\mathbf{A}}_i, \mathbf{x}^\circ_{ii}, \tilde{\mathbf{c}}_i)$] is a minimal realization of the causal part of $G_i^{-1}(s)\mathbf{c}_{ij}(s\mathbf{I} - \mathbf{A}_{ij})^{-1}\mathbf{x}^\circ_{ij}$ $[G_i^{-1}(s)\mathbf{c}_i(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{x}^\circ_i]$. Finally, we can conclude that

$$\|\zeta_i\|^{t_{ri}}_\infty \le \sum_{j=1,N} \|\hat{g}_i * g_{ij}\|_1 \left(\|u_j\|^{t_{ri}}_\infty + \|\nu_j\|^{t_{ri}}_\infty \right) + V_i$$

Introducing the notation

$$\begin{aligned} \mathbf{k} &= [k_1, \ldots, k_N]^T, \\ \mathbf{V} &= [V_1, \ldots, V_N]^T, \\ \mathbf{U} &= [U_1, \ldots, U_N]^T, \\ \mathbf{N} &= [N_1, \ldots, N_N]^T, \\ \epsilon &= [\epsilon_1, \ldots, \epsilon_N]^T \end{aligned}$$

and

$$\mathbf{P} = \{P_{ij}\} = \{\|\hat{g}_i(t) * g_{ij}(t)\|_1\}, \qquad (6)$$

the sliding condition (5) can be written as

$$\mathbf{k} = \mathbf{P}\mathbf{k} + (\mathbf{I} + \mathbf{P})\mathbf{N} + \mathbf{U} + \mathbf{V} + \epsilon \qquad (7)$$

Based on the previous development, and under the assumptions of lemma 1, we can state the following

Theorem 1 *The following conditions are equivalent:*

- *There exists a decentralized variable structure control of type (4) that guarantees the existence of a robust sliding regime on any assigned linear manifold* $\eta_i z_i(t) = 0$ *such that* $\eta_i \bar{\mathbf{b}}_i \ne 0$;
- *the spectrum of* $\mathbf{P}$ *in (6) is strictly within the unit disc in* $\mathbf{C}$.
- $\lambda_{PF}(\mathbf{P}) < 1$, *where* $\lambda_{PF}(\mathbf{P})$ *is the Perron-Frobenius eigenvalue of* $\mathbf{P}$;
- *there exists an induced norm* $\|\cdot\|$ *on* $\mathbb{R}^{n \times n}$ *such that* $\|\mathbf{P}\| \le 1$;
- $\mathbf{W} = \mathbf{I} - \mathbf{P}$ *is an M-matrix.*

Proof. Conditions above regard the existence of a nonnegative $\mathbf{k}$ solving (7) for nonnegative $\mathbf{U}$, $\mathbf{Z}$,$\mathbf{N}$ and ϵ such that $(\mathbf{I}+\mathbf{P})\mathbf{N}+\mathbf{U}+\mathbf{Z}+\epsilon \ne 0$, and follow directly from the theory of positive matrices (see e.g. [Gantmacher, 1977]).
□.

Remark 1. Under the conditions of theorem 1, any $\mathbf{k}$ belonging to the cone in $\mathbb{R}^n$ with vertex in $(\mathbf{I}-\mathbf{P})^{-1}((\mathbf{I}+\mathbf{P})\mathbf{N}+\mathbf{U}+\mathbf{Z})$ and positively spanned by the columns of $(\mathbf{I}-\mathbf{P})^{-1}$, except the vertex, corresponds to a set of amplitudes of discontinuous control actions that maintains sliding.

Remark 2. Note that easy-to-check sufficient conditions for (7) to have nonnegative solutions are derived from Gershgorin's theorem as

$$\|\mathbf{P}\|_\infty < 1 \qquad \text{11Pr1} < 1 \qquad (8)$$

i.e., in terms of conventional row or column dominance. Note also that, according to the theory of generalized diagonal dominance (see e.g. [Araki and Nwokah, 1975]), conditions in theorem 1 guarantee the existence of an input-output scaling matrix $\mathbf{S}$ with positive elements such that $\mathbf{S}^{-1}\,\mathbf{PS}$ satisfies one of the (8). Conditions (8) can be stated in terms of norms induced on the space of linear $N \times N$ operators by L_∞ and L_1 norms on the space of input signals

$$\|\mathbf{I} - \mathbf{G}_D^{-1}\,\mathbf{G}\|_{L_\infty} < 1 \qquad \|\mathbf{I} - \mathbf{G}_D^{-1}\,\mathbf{G}\|_{L_1} < 1,$$

that show the intrinsic significance of the conditions.

Remark 3. Recall from [Ohta, Siljak, and Matsumoto, 1980] that, for a multivariable plant $\mathbf{G}(s)$ with diagonally decentralized linear feedback controller $K(s)$, the dominance condition is written

$$\rho_{PF}(\hat{\mathbf{P}}(s)) < 1 \quad \forall s \in \mathcal{D} \qquad (9)$$

where $\mathcal{D}$ is the Nyquist contour, and

$$\hat{\mathbf{P}}(s) = \left\{ \left\| G_{ij}(s)\mathbf{K}_j(s)[\mathbf{I} + G_j(s)\mathbf{K}_j(s)]^{-1} \right\| \right\}$$

A particularly interesting case is when dominance holds for arbitrarily large, constant feedback gains $\mathbf{K}_i$, in which case (9) is replaced by

$$\rho_{PF}(\hat{\mathbf{P}}'(s)) < 1 \quad \forall s \in \mathcal{D}$$

where

$$\hat{\mathbf{P}}(s)' = \left\{ \left\| G_{ij}(s)G_j(s)^{-1} \right\| \right\}$$

Note that, in the case of no uncertainty on the diagonal subsystems, under hypotheses **H1** and **H2** we have

$$\begin{aligned} \hat{P}'_{ij}(s) &= \left| \int_0^\infty \hat{g}_{jj}(\tau) * g_{ij}(\tau) \exp\,(-\iota\omega\tau) d\tau \right| \le \\ &\le \int_0^\infty |\hat{g}_{jj}(\tau) * g_{ij}(\tau)| \, d\tau = \mathbf{P}_{ij}. \end{aligned}$$

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Hence, from the theory of nonegative matrices,

$$\rho_{PF}(\mathbf{P}) \geq \rho_{PF}(\hat{\mathbf{P}}')$$

Note that the frequency–domain dominance condition is less restrictive than the corresponding time–domain condition for the existence of a DVSC, in agreement with the fact that the latter ensures the possibility of imparting an arbitrary behaviour to the output, while the same can not be afforded by static output feedback.

Remark 4. It may be noted that, although we assume an input–output description of the system, the control law (4) employs states of a realization of transfer functions on the diagonal. This involves in general the necessity of setting up observers with suitable dynamics to reject input disturbances $\zeta_i(t)$. Nonlinear, variable structure observers have been proposed in the literature that can be applied in principle to this problem (see for instance [Slotine, Hedrick, and Misawa, 1987] and [Walcott and Zak, 1988]).

3 Applications

Results of previous section have important implications in practical applications. The one described in this paper is to the synthesis of a tracking controller for a MIMO, time–domain dominant system in input–output form. One further hypothesis is added on the system, i.e. that diagonal systems $G_i(s)$ have relative degree one. This helps in simplifying the control scheme with regard to the state observation problems discussed in remark 4 above. Note that this assumption is less conservative than the strict positive realness assumption necessary to use sliding–mode observers described e.g. by Walcott and Zak [1988].

The proposed control scheme is based on N reference models $(\mathbf{A}_i, \mathbf{b}_i, \mathbf{c}_i)$, corresponding to minimal realizations of $G_i(s)$, with inputs, states, and outputs $\hat{u}_i$, $\hat{\mathbf{z}}_i$, and $\hat{w}_i$, respectively (see fig. 1).

The desired trajectory to be followed is described by

$$\begin{cases} \dot{\mathbf{r}}_i(t) &= \mathbf{A}_{ri}\mathbf{r}_i(t) + \mathbf{b}_{ri}\mathbf{v}_{ri}(t), \ \mathbf{r}(0) = \mathbf{r}_o \\ y_{ri}(t) &= \mathbf{c}_i\mathbf{r}_i(t) \end{cases}$$

Consider the linear manifold

$$\mathcal{Z}_i' : \sigma_i \stackrel{def}{=} \gamma_i\left(\hat{\mathbf{z}}_i(t) - \mathbf{r}_i(t)\right) = 0$$

where $\gamma_i \in \mathbb{R}^{1\times n_i}$ is chosen such that $\gamma_i\mathbf{b}_i = 1$. The control law

$$\hat{u}_i(t) = -\gamma_i\mathbf{A}_i\hat{\mathbf{z}}_i(t) - \hat{k}_i \mathsf{sign}\ \sigma_i$$

with

$$\hat{k}_i = \|\gamma_i\ \dot{\mathbf{r}}_i\|_\infty + \hat{\epsilon}_i.$$

with $\hat{\epsilon}_i \in \mathbb{R}_+$, guarantees that $\sigma_i(t) = 0$ for all $t > t_{\sigma i} = |\sigma_i(0)|/\epsilon_i$. After the onset of such sliding motion, states $\hat{\mathbf{z}}_i$ converge to $\mathbf{r}_i$ with the dynamics imposed by the choice of γ_i, and consequently $\hat{w}_i$ asymptotically tracks $\mathbf{y}_{ri}$.

Consider now the system $\mathcal{S}_{z_i}$ described by (1), whose output w_i coincides with the i–th output channel of the plant in the hypotheses of lemma 1. Since

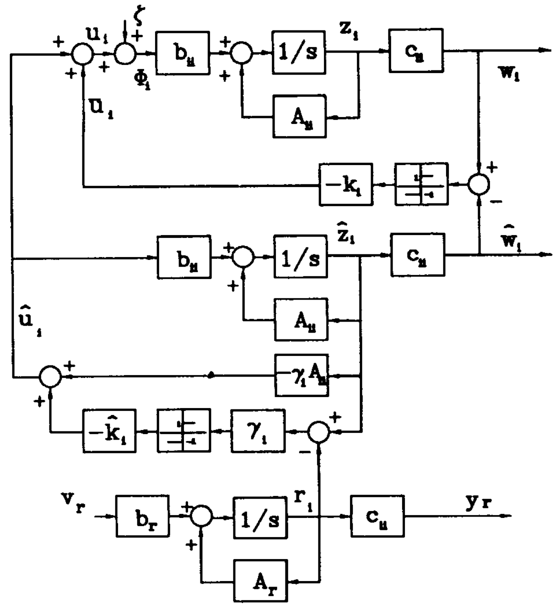

Figure 1: The i–th channel of the proposed control scheme. Note that $\zeta' = \nu_i + \zeta_i$ represents the sum of input disturbances and model inaccuracies as reduced to inputs.

input disturbances $\mathbf{b}_i\zeta_i(t)$ comply by definition with matching conditions for disturbance rejection, the next design step consists in setting up one further control loop to have $w_i(t) \equiv y_i(t)$ effectively track $\hat{w}_i(t)$ (and hence $y_{ri}(t)$). As already noted, states $\mathbf{z}_i$ of $\mathcal{S}_{z_i}$ are not accessible neither can they be reconstructed since $\zeta_i(t)$ is not known a priori. In the assumption that $G_i(s)$ has relative degree one, however, a linear manifold

$$\begin{aligned} \mathcal{Z}_i'' : \varsigma_i &\stackrel{def}{=} \mathbf{c}_i\mathbf{e}_i = 0, \\ \mathbf{e}_i(t) &\stackrel{def}{=} \mathbf{z}_i(t) - \hat{\mathbf{z}}_i(t), \end{aligned}$$

can be chosen such that an equivalent control is defined on it as

$$u_{eq}''(t) = -\frac{\mathbf{c}_i\mathbf{A}_i\mathbf{e}_i(t)}{\mathbf{c}_i\mathbf{b}_i} - \zeta_i(t) + \hat{u}_i(t).$$

From hypothesis **H1**, moreover, sliding motion on $\mathcal{Z}_i''$ is asymptotically stable. Note that, although the dynamics of sliding motion on $\mathcal{Z}_i''$ can not be modified by design, they have no influence on the characteristics of tracking, since they are not observable from the plant output.

The sliding condition $\varsigma(t)\dot{\varsigma}(t) \leq 0$ can be met by applying the control signal

$$u_i(t) = \hat{u}_i(t) + \bar{u}_i(t)$$

with $\bar{u}_i(t) = \bar{k}_i \mathsf{sign}\ \varsigma_i(t) = \bar{k}_i \mathsf{sign}\ (y_i - \hat{w}_i)$ and

$$\bar{k}_i = (\mathbf{I} - \mathbf{P})^{-1}(\mathbf{U}' + \mathbf{Z} + \bar{\epsilon}_i),$$

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where $\bar{\epsilon}_i \in \mathbb{R}_+$ and the elements of $\mathbf{U}'$ consist of bounds for $\left\| \frac{\mathbf{c}_i \mathbf{A}_i \mathbf{e}_i}{\mathbf{c}_i \mathbf{b}_i} \right\|_\infty^{t_{si}}$.

4 Simulation results

In this section, some simulation results relative to a 4 inputs,4 outputs system are presented. The plant is

$$\mathbf{G}(s) = \begin{bmatrix} G_1(s) + \frac{.2}{(s+2)} & \frac{(s+10)}{(s+4)(s+12)} & \cdots \\ \frac{1}{(s+3)(s+5)} & G_2(s) + \frac{0.1}{(s+2)} & \\ \frac{(s+4)}{(s+6)(s+8)} & \frac{1}{(s+3)(s+5)} & \\ \frac{(s+10)}{(s+4)(s+12)} & \frac{(s+4)}{(s+6)(s+8)} & \end{bmatrix}$$

$$\begin{matrix} \frac{(s+7)}{(s+6)(s+8)} & \frac{1}{(s+3)(s+5)} \\ \frac{(s+10)}{(s+4)(s+12)} & \frac{(s+4)}{(s+3)(s+6)} \\ G_3(s) + \frac{0.1(s+4)}{(s+2)(s+3)} & \frac{(s+10}{(s+4)(s+12)} \\ \frac{1}{(s+3)(s+5)} & G_4(s) + \frac{0.2}{(s+3)} \end{matrix} \Bigg]$$

where the only known elements are the diagonal ones,

$$\begin{aligned} G_1 &= \frac{10(s+2)}{(s-3)(s-5)} \\ G_2 &= \frac{6(s+3)}{(s+5)(s+7)} \\ G_3 &= \frac{15(s+4)}{(s-5)(s-15)} \\ G_4 &= \frac{5(s+3)}{(s+2)(s+7)}. \end{aligned}$$

The time–domain dominance condition evaluated on

$$\mathbf{P} = \begin{bmatrix} 0.1414 & 0.4557 & 0.3444 & 0.0726 \\ 0.1296 & 0.0972 & 0.4051 & 0.1885 \\ 0.2721 & 0.0681 & 0.0795 & 0.3889 \\ 0.2291 & 0.3227 & 0.0646 & 0.0660 \end{bmatrix}$$

provides $\rho_{PF}(\mathbf{P}) = 0.8311$. Input disturbances on the four channels are bounded in amplitude and frequency by 7 and 2000 rad/sec, respectively. The output trajectories to be tracked are generated for each channel by filtering the synusoidal inputs

$$\begin{aligned} v_{r1} &= 50\sin(60t) \\ v_{r2} &= 20\cos(100t) \\ v_{r3} &= 30\sin(40t) \\ v_{r4} &= 60\cos(80t) \end{aligned}$$

through a second–order filter with coincident poles in -30. The sliding manifolds $\mathcal{Z}_i$ have been chosen so as to minimize an LQ index with identity state weight matrix and 0.01 output weight, giving $\gamma_1 = [100.5\ 1]$, $\gamma_2 = [59.6\ 1]$, $\gamma_3 = [150.3\ 1]$, and $\gamma_4 = [49.3\ 1]$. For the given refence dynamics and initial conditions, discontinuous control gains $\hat{k}_1 = 101.5 > \|\gamma \dot{\mathbf{r}}_1\|_\infty = 84.6$, $\hat{k}_2 = 31.9 > \|\gamma \dot{\mathbf{r}}_2\|_\infty = 26.6$, $\hat{k}_3 = 100.9 > \|\gamma \dot{\mathbf{r}}_3\|_\infty = 84.1$, and $\hat{k}_4 = 81.1 > \|\gamma_4 \dot{\mathbf{r}}_4\|_\infty = 67.6$ are chosen. Initial conditions of the off-nominal systems have been assumed to be contained in balls of radius 0.05. Accordingly, the width of the discontinuous part of the second variable structure control loop are chosen as $\bar{\mathbf{k}} = [\ 64.8 \quad 50.1 \quad 55.1 \quad 53.0\ ]$. The output of the plant and of the reference model are compared with the desired trajectory in fig. 2 a,b,c, and d, for channels 1,2,3,and 4, respectively. Time scales are in seconds. The behaviour of the controlled output system is quite satisfactory. Input signals are reported in fig. 3. Excessive chattering in the control inputs could be eliminated by suitable smoothing techniques for VS controllers, such as the "boundary layer" method of Slotine and Sastry [1983].

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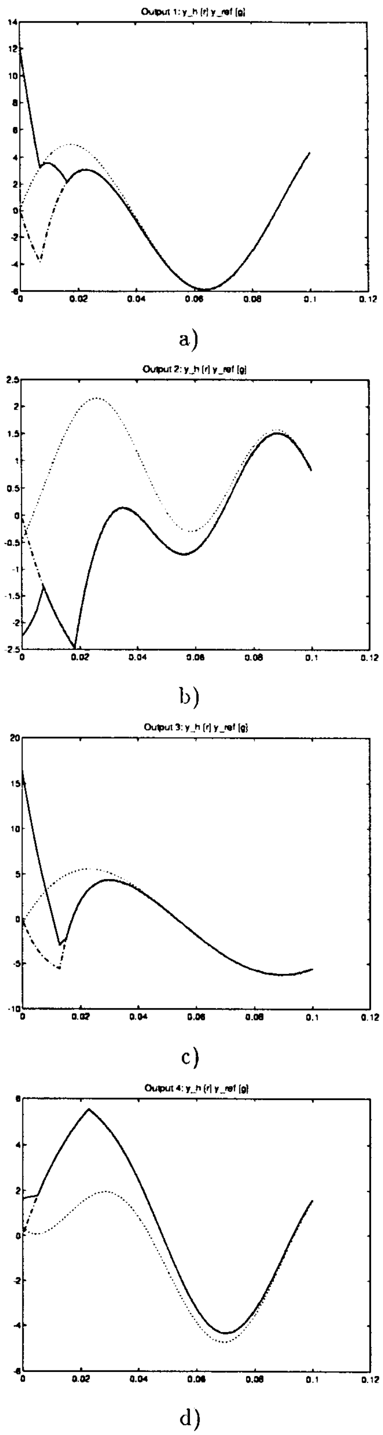

Figure 2: Desired trajectory (dotted) , reference model (dash–dot), and plant output (continuous) for the four channels

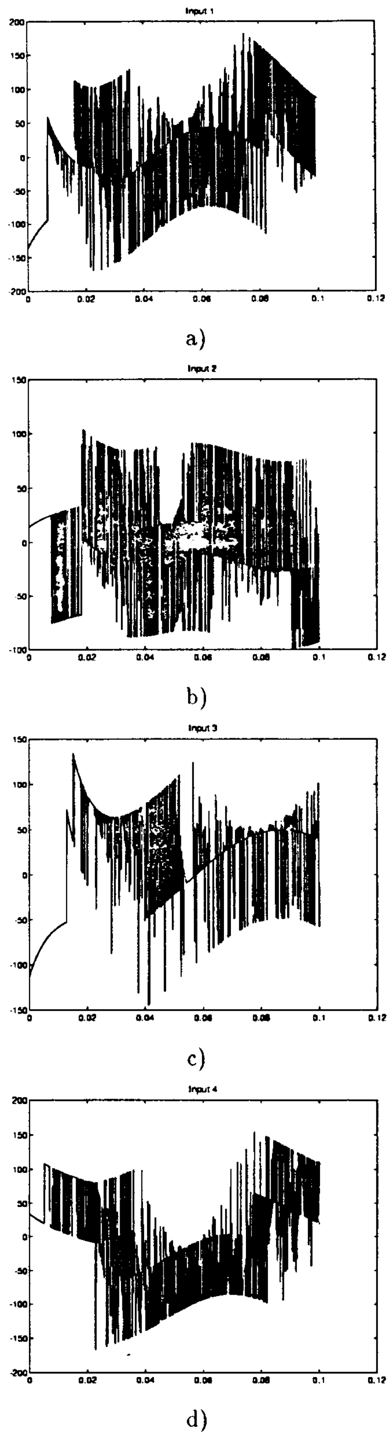

Figure 3: Variable-structure decentralized control input signals for the four channels

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