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Dexterity Through Rolling: Manipulation of Unknown Objects

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Abstract

The nonholonomy exhibited by kinematic systems consisting of bodies rolling on top of each other can be used to the purpose of building dexterous mechanisms with a minimum hardware complication. Such desirable an engineering feature can be fully exploited, however, only if the capability of planning and controlling rolling motions of arbitrary objects is achieved. In this paper we present recent advances of both theoretical and experimental nature towards realizing a robot gripper for manipulation of objects whose shape is not known a priori, but is reconstructed as manipulation proceeds.

1 Introduction

Few recent works in mechanism design and robotics reported on the possibility of exploiting nonholonomic mechanical phenomena in order to design devices that achieve complex tasks with a reduced number of actuators ([8], [19], [21], [5], [10]). Although this seems to be a promising new approach to reducing the complexity, cost, weight, and unreliability of the hardware used in such devices, it is true in general that planning and controlling nonholonomic systems is more difficult than holonomic ones. Indeed, notwithstanding the large efforts spent by applied mathematicians, control engineers, and roboticists on the subject, many open problems remain unsolved at the theoretical level, and even more at the computational and implementation level.

In this paper we report on some results that have been obtained in the study of manipulation of objects by rolling, in view of the realization of a robot gripper that exploits rolling to achieve dexterity, i.e., the ability to arbitrarily change the location and orientation of the manipulated objects. A first prototype of such device was presented by [5], along with some preliminary experiments in planning and controlling motions of a sphere manipulated by rolling. Marigo *et al.* [17] applied manipulation by rolling to objects of polyhedral shape. The design of grippers exploiting rolling was based on the conjecture that a kinematic system comprised of *almost any* pair of rolling surfaces is controllable, which has been shown true recently by [16]:

Theorem 1 [Controllability of rolling bodies] *The kinematic constraint of rolling without sliding between smooth rigid bodies is*

a) holonomic if and only if bodies are the mirror image of each other;

b) maximally nonholonomic (hence completely controllable) otherwise.

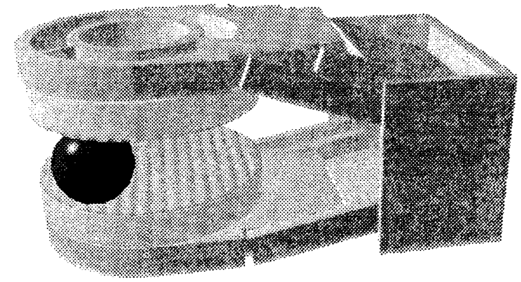

Figure 1: The second generation dexterous gripper (DxGrip-II) designed and built at Centro "E. Piaggio", University of Pisa.

As an obvious corollary, any convex object rolling between two flat fingers is completely controllable from any initial to any final desired configuration. Once thus established the theoretical possibility of building a hand that can achieve arbitrary relocation and orientation of manipulated objects by rolling them between fingers, its practical realizability depends on the possibility of planning and controlling such motions.

In this paper we turn our attention to manipulation by rolling of objects with arbitrary regular (i.e., smooth) surface, facing the case that the object shape is not known a priori, but only perceived through tactile sensing, as it often happens in real-world applications.

2 Exploration of Unknown Objects

As already mentioned, parts to be manipulated are sometimes not known a priori to the robot, and information on their shape need to be gathered before manipulation can be planned and executed. In this section we describe the means by which it is possible to elicit shape information from rolling, with particular reference to the case of regular surfaces.

The dextrous gripper used in our experiments consists of two parallel plates, whose motions are actuated by four electrical motors (see section 4 for a description). The procedure used to reconstruct the

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surface of unknown objects is as follows:

i) The hand (with fingers open) is put around the object to be explored, and then closed in guarded mode with a contact force threshold;

ii) While actuators commanding the distance between the fingers regulates a suitable grasping force to avoid slippage of the object on the fingers, the actuators that command relative rotations and translations of the fingers follow random trajectories causing the object to roll between the fingers;

iii) the position of the contact point on the surface of the upper and lower fingers, as well as the position and velocity of the gripper joints, are measured during exploration; this information is used to calculate the position and velocity of the contact points on the object surface.

In order to control the grasping force, a six-axis force/torque sensor is used on the fingers. To detect the location of contact points on the fingers, the same sensors can be used in conjunction with the "intrinsic" tactile sensing algorithms described in [4], which also applies to fingers with a general convex surface.

To reconstruct an approximation of the surface of the object, it is necessary to evaluate the instantaneous position of the contact points with respect to a cartesian frame fixed with the object. Let the origin of this frame be denoted by $\mathbf{o}$, and let three unit vectors parallel to the x, y, and z axes of the body frame be denoted by $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, respectively (see fig.2). Let the object surface be described in spherical coordinates, i.e., the position of a generic point (except the north and south poles) of the surface in the body-fixed frame is given in terms of azimuth $u \in [-\pi, \pi)$ and elevation $v \in (-\pi/2, \pi/2)$ angles as

$$\begin{cases} x = \rho(u,v)\cos v \cos u \\ y = \rho(u,v)\cos v \sin u \\ z = \rho(u,v)\sin v \end{cases} \tag{1}$$

where $\rho(u, v)$ is a continuous function of the azimuth and elevation u, v. Notice that spherical coordinates are convenient for several reasons, among which is the fact that they provide an orthogonal parametrization of all surfaces of revolution (i.e., surfaces with an axis of symmetry), except at their poles. For surfaces of revolution, $\rho_u \stackrel{def}{=} \frac{\partial \rho(u,v)}{\partial u} = 0$. The position of the contact points on the upper and lower finger (denoted by $\mathbf{c}_1$ and $\mathbf{c}_2$, respectively) being known from tactile sensing, their velocities $\dot{\mathbf{c}}_1$ and $\dot{\mathbf{c}}_2$ with respect to a fixed wrist frame can be easily calculated by using the finger Jacobian matrix and measures of finger joint velocities. From data on the position and velocity of two points on the rigid object being manipulated, and using assumptions on friction at the contacts, one easily obtains the instantaneous angular velocity $\boldsymbol{\omega}$ of the rolling object in the wrist frame.

Letting $\mathbf{o}$ and $\mathbf{R} = [\mathbf{i}\ \mathbf{j}\ \mathbf{k}]$ denote the position of the origin and the orientation matrix of the reference frame fixed to the rolling object, the object motion is

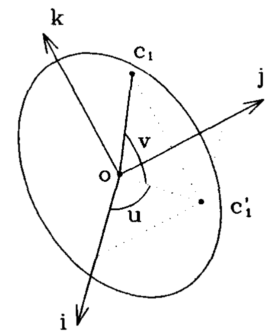

Figure 2: Spherical coordinates on the manipulated object.

described by the following differential equations:

$$\begin{aligned} \dot{\mathbf{o}} &= \dot{\mathbf{c}}_1 + \boldsymbol{\omega} \times (\mathbf{o} - \mathbf{c}_1) \\ \dot{\mathbf{R}} &= \boldsymbol{\omega} \times \mathbf{R} \end{aligned}$$

Integrating these equations during the exploration time, the instantaneous position and orientation of the body can be obtained. From geometric considerations (see fig.2) we obtain at each time t the desired information on the coordinates of two points of the object surface from tactile sensor measurements $\mathbf{c}_1(t)$ and $\mathbf{c}_2(t)$ from (1) by setting for $i = 1, 2$

$$\begin{aligned} v_i(t) &= \arcsin \frac{(\mathbf{c}_i - \mathbf{o})^T \mathbf{k}}{\rho_i}; \\ u_i(t) &= \mathrm{atan2}\ \left((\mathbf{c}'_i - \mathbf{o})^T \mathbf{j},\ (\mathbf{c}'_i - \mathbf{o})^T \mathbf{i} \right); \\ \rho_i(t) &= \|\mathbf{c}_i - \mathbf{o}\|, \end{aligned}$$

where

$$\mathbf{c}'_i = \mathbf{c}_i - \rho_i \sin v_i \mathbf{k}.$$

The problem of reconstructing a surface from knowledge of a number of its points is an important issue common to several fields of science and engineering. In robotics, the problem has been studied extensively in relation with processing data from cameras, range finders, and/or tactile sensors. Part of the literature is concerned with the "object recognition", or model matching problem (see e.g. [12], [15], [13], [11]). Works concerned with shape reconstruction deal with fitting experimental data with general models of surfaces (see e.g. [7], [14]). Various methods are distinguished by the information used and the surface model adopted to fit data. Allen [1] used bicubic (Coons') patches to fit data from vision and touch sensors, while [2] used superquadrics; [3] approximated objects by surfaces of revolution, and were able to determine their axis of symmetry by using tactile measurement of contact points, contact normals, and curvatures at the contact points; [9] considered haptic recognition of objects based on polyhedral shape approximations.

With respect to the existing literature, where surface reconstruction is mostly intended for object recognition, the problem we consider is to gather the surface information necessary to obtain sufficiently accurate formulae for the control vector fields appearing in the differential equation of rolling. As these vector fields are computed through differential operations from the surface description, it is necessary not only that the reconstruction is given in terms of analytic functions which are defined on as large a domain as possible, but also are sufficiently smooth to avoid noise amplification through differentiation.

In order to master completely the accuracy/smoothness tradeoff in reconstruction, we found tools from regularization theory to be most effective (see e.g. [22] and [23]). In that framework, the problem of finding the "best" function approximating a multivariate function $y(\mathbf{x})$, whose values y_i at k points $\mathbf{x}_i$ are known (although with errors), is formulated as the minimization of the variational expression

$$H(f) = \sum_{i=0}^{k} (y_i - f(\mathbf{x}_i))^2 + \lambda \|Pf\|^2 \qquad (2)$$

where P is a differential operator used to weigh the "bumpiness" of the approximating function, and λ is a regularization parameter, that controls the compromise between the degree of smoothness of the solution, and its closeness to data ([20]). Such standard regularization technique provides solutions that are equivalent to generalized splines: for example, for single variable functions, it can be shown that with the differential operator

$$\|Pf\|^2 = \int_R \left[\frac{\partial^2 f(x)}{\partial x^2} \right]^2 \partial x$$

the solution of the regularization problem is given by cubic splines. In general, solution of (2) leads to the associated Euler-Lagrange equation

$$\hat{P}Pf(x) = \frac{1}{\lambda} \sum_{i=0}^{k} (y_i - f(\mathbf{x}))\delta(\mathbf{x} - \mathbf{x}_i) \qquad (3)$$

where $\hat{P}$ is the adjoint operator of P and δ is the Dirac delta function. The solution of (3) can be written as

$$f(\mathbf{x}) = \frac{1}{\lambda} \sum_{i=0}^{k} (y_i - f(\mathbf{x}_i)) G(\mathbf{x}; \mathbf{x}_i) \qquad (4)$$

where $G(\mathbf{x}; \mathbf{x}_i)$ are the Green functions of the differential operator $\hat{P}P$. Green functions are actually *radial* functions of their arguments $G(\mathbf{x}; \mathbf{y}) = G(\|\mathbf{x} - \mathbf{y}\|)$ when P is rotationally and translationally invariant. In such case, the solution of the regularization problem is a sum of *radial basis functions*:

$$f(\mathbf{x}) = \sum_{i=0}^{k} c_i G(\|\mathbf{x} - \mathbf{x}_i\|), \qquad (5)$$

where the weights c_i can be evaluated by simple linear algebraic operations. Some commonly encountered radial basis functions used in regularization theory and in the closely allied field of neural networks are

$$G(r) = \begin{cases} r & \text{(linear interpolation)} \\ r^3 & \text{(cubic interpolation)} \\ \sqrt{r^2 + c^2} & \text{(multiquadric)} \\ \frac{1}{\sqrt{r^2 + c^2}} & \text{(inverse multiquadric)} \\ e^{-\frac{r^2}{\sigma^2}} & \text{(gaussian)} \end{cases}$$

The problem of reconstructing a surface described in spherical coordinates (1) amounts to approximating a smooth function $\rho : S^2 \to \mathbb{R}$, $\rho = \rho(u, v)$ of the azimuth and elevation angles u, v, for which a set of points $\rho(u_i, v_i) = \rho_i$ are given from exploration data. With respect to the theory above resumed, the fact that the domain manifold S^2 is not globally equivalent to $\mathbb{R}^2$ imposes some modifications in the choice of basis functions. Following [23], we choose

$$\rho = \sum_{l=0}^{n} \sum_{s=-l}^{l} f_{ls} Y_{ls} \qquad (6)$$

where f_{ls} are coefficients, and Y_{ls} are the eigenfunctions of the (surface) Laplacian on the sphere, i.e. the *spherical harmonics*, whose expression in coordinates is

$$Y_{ls}(u, v) = \begin{cases} U_{ls} \cos(us) P_l^s(\sin v) & 0 < s \leq l \\ U_{ls} \sin(us) P_l^{|s|}(\sin v) & -l \leq s < 0 \\ U_{l0} P_l(\sin v) & s = 0 \end{cases} \qquad (7)$$

for $l = 0, 1, \ldots$. Here,

$$\begin{aligned} U_{ls} &= \sqrt{2}\sqrt{\tfrac{2l+1}{4\pi}\tfrac{(l-|s|)!}{(l+|s|)!}} \quad & s \neq 0 \\ &= \sqrt{\tfrac{2l+1}{4\pi}} & s = 0 \end{aligned}$$

P_l, $l = 0, 1, \ldots$, are the Legendre polynomials, and P_l^s are the Legendre functions

$$P_l^s(z) = (-1)^s (1 - z^2)^{\frac{s}{2}} \frac{\partial^s}{\partial z^s} P_l(z)$$

Notice that Y_{l0} are surfaces of revolution. The unknown coefficients are obtained by minimizing the regularized spherical least–squares functional

$$\begin{aligned} H(\lambda) \;\; = \;\; & \frac{1}{n} \sum_{i=1}^{k} \left(\rho_i - \textstyle\sum_{l=0}^{n} \sum_{s=-l}^{l} f_{ls} Y_{ls}(u_i, v_i) \right)^2 \\ & + \lambda \sum_{l=0}^{n} \sum_{s=-l}^{l} [l(l+1)]^m f_{ls}^2 \end{aligned} \qquad (8)$$

Arranging the index set $\{(l, s)\}$ in a convenient order, and letting $\mathbf{f}$ be the vector of f_{ls} and $\mathbf{X}$ be the matrix with $(i, ls)_{th}$ entry $Y_{ls}(u_i, v_i)$, (8) becomes

$$\frac{1}{n} \|\mathbf{y} - \mathbf{X}\mathbf{f}\|^2 + \lambda \mathbf{f}^T \mathbf{D} \mathbf{f}$$

where $\mathbf{D}$ is the diagonal matrix with $(ls, ls)_{th}$ entry $[l(l+1)]^m$. The minimizing vector $\mathbf{f}_\lambda$ is simply obtained by solving the following linear system of equations,

$$\mathbf{f}_\lambda = (\mathbf{X}^T \mathbf{X} + \lambda \mathbf{D})^{-1} \mathbf{X}^T \mathbf{y}.$$

3 Planning for General Surfaces

The kinematics of rolling surfaces are a well-known example of driftless nonholonomic system. Generally speaking, the problem of planning a driftless system

$$\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\mathbf{u},\ \mathbf{q}(0) = \mathbf{q}_0 \in \mathbb{R}^n, \tag{9}$$

consists in finding, for each pair $(\mathbf{q}_0, \mathbf{q}_f)$, a control function $\mathbf{u} : [0,1] \to \mathbb{R}^m$, $t \mapsto \mathbf{u}(t)$ within an admissible set U such that, for the corresponding solution $\mathbf{q}(t, \mathbf{q}_0, \mathbf{u})$ of (9), it holds $\mathbf{q}(1, \mathbf{q}_0, \mathbf{u}) = \mathbf{q}_f$. A brute force approach to this problem consists in 1) solving (9) for a generic input $\mathbf{u}(\mathbf{p})$ in a sufficiently general family $\bar{U} \subset U$ suitably parameterized by $\mathbf{p} \in \mathbb{R}^p$, and 2) solve the set of n nonlinear equations $\mathbf{q}(1, \mathbf{x}_0, \mathbf{p}) = \mathbf{q}_f$ in the p unknowns $\mathbf{p}$. Obviously, both steps possibly hide enormous difficulties, as solving an O.D.E. in "closed" form is rarely possible, and solving large systems of nonlinear equations is notoriously hard. The fact, shown in [6], that the equations of motion for a convex body rolling between flat fingers can be put in strictly triangular form tremendously help in this regard.

The relevance of strict triangular forms to planning is twofold. In fact, an O.D.E. in strictly triangular form can be easily solved by quadratures, i.e. the flow of the control vector fields is found simply by subsequently integrating their components over time. Furthermore, strict triangularity allows to break the solution of the system of n nonlinear equations of step 2) of the generic algorithm above, into the solution of multiple systems of fewer equations in fewer unknowns.

These advantages of the triangular form are exploited in the following algorithm. For objects with an axis of symmetry, which we consider henceforth for simplicity, the strictly triangular form reads as (see [6])

$$\dot{\mathbf{z}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\rho \sin v - \rho_v \cos v}{\sqrt{\rho^2+\rho_v^2}} & 0 \\ \rho \cos v \cos \psi & -\sqrt{\rho^2+\rho_v^2}\sin\psi \\ \rho \cos v \sin \psi & -\sqrt{\rho^2+\rho_v^2}\sin\psi \end{bmatrix} \hat{\mathbf{w}}, \tag{10}$$

where $\mathbf{z} = [u, v, \psi, x, y]$. A possible choice for the admissible input set U is to consider piecewise constant inputs over time intervals T with an alternating pattern, i.e.

$$\hat{\mathbf{w}}(t = kT + \tau) = \hat{\mathbf{w}}_k = \begin{cases} [\lambda_k, 0]^T & \text{k even} \\ [0, \lambda_k]^T & \text{k odd} \end{cases},$$

for $0 \le \tau < T$, and $k = 0, 1, \ldots, N-1$, such that the flows $\Phi_{\mathbf{g}_i}^{\lambda_k T}$ of the two control vector fields are followed sequentially N times. The flows can be integrated explicitly starting from initial conditions $\mathbf{z}_k = \mathbf{z}(t = kT)$ for k even as

$$\mathbf{z}_{k+1} = \begin{bmatrix} \lambda_k T + u_k \\ v_k \\ \Gamma_k \lambda_k T + \psi_k \\ \frac{\rho \cos v_k}{\Gamma_k}(\sin\psi_{k+1} - \sin\psi_k) + x_k \\ \frac{\rho \cos v_k}{\Gamma_k}(\cos\psi_{k+1} - \cos\psi_k) + y_k. \end{bmatrix} \tag{11}$$

where

$$\Gamma_k = \left.\frac{\rho \sin v - \rho_v \cos v}{\sqrt{\rho^2+\rho_v^2}}\right|_{\mathbf{z}=\mathbf{z}_k},$$

and for k odd as

$$\mathbf{z}_{k+1} = \begin{bmatrix} u_k \\ \lambda_k T + v_k \\ \psi_k \\ -\lambda_k \sin\psi_k \Delta_k + x_k \\ -\lambda_k \cos\psi_k \Delta_k + y_k \end{bmatrix} \tag{12}$$

with

$$\Delta_k = \int_{kT}^{(k+1)T} \sqrt{\rho^2+\rho_v^2}\, dt\,.$$

In terms of these definitions, the planning problem can be restated as:

Problem 1 *Given a pair $(\mathbf{z}_s, \mathbf{z}_f)$, find an integer N and an N-tuple of real numbers $(\lambda_0, \ldots, \lambda_{N-1})$ such that the nonlinear, discrete-time system defined by (11), (12), with $\mathbf{z}_0 = \mathbf{z}_s$, has $\mathbf{z}_N = \mathbf{z}_f$.*

A solution to this problem is provided by the following algorithm:

Step 1) Apply first inputs that take the first two variables to the desired value: set $\lambda_0 = \mu_0 = (u_f - u_0)/T$, $\lambda_1 = \mu_1 = (v_f - v_0)/T$, such that $\mathbf{z}(2T) = [u_f, v_f, \psi_2, x_2, y_2]$;

Step 2) Apply a sequence of five inputs that does not alter the first two variables, i.e. $(\lambda_2 = 0, \lambda_3 = \mu_2, \lambda_4 = \mu_3, \lambda_5 = -\mu_2, \lambda_6 = -\mu_3)$ (the void input is included for preserving index parity). By choosing

$$\mu_3 = \frac{\psi_f - \psi_2}{(\Gamma_4 - \Gamma_6)T},$$

with μ_2 arbitrary (provided that $\Gamma_4 \neq \Gamma_2$), the third variable reaches its desired value: $\mathbf{z}(6T) = [u_f, v_f, \psi_f, x_6, y_6]$;

Step 3) Apply a sequence of 15 controls that does not alter the first three variables, namely ($\lambda_7 = 0, \lambda_8 = \mu_4, \lambda_9 = \mu_5, \lambda_{10} = -\mu_4, \lambda_{11} = -\mu_5 + \mu_6, \lambda_{12} = \mu_7, \lambda_{13} = -\mu_6, , \lambda_{14} = -\mu_7, \lambda_{15} = \mu_5, \lambda_{16} = \mu_4, \lambda_{17} = -\mu_5, \lambda_{18} = -\mu_4 + \mu_7, \lambda_{19} = \mu_6, \lambda_{20} = -\mu_7, \lambda_{21} = -\mu_6$. For such a sequence to take the last two variables to their desired value, any quadruple $(\mu_4, \mu_5, \mu_6, \mu_7)$ solving the system of two nonlinear algebraic equations $x_{22}(x_6, \mu_4, \mu_5, \mu_6, \mu_7) = x_f; y_{22}(y_6, \mu_4, \mu_5, \mu_6, \mu_7) = y_f$ can be chosen.

Remark 1. The algorithm description highlights the role of commutator sequences of type $(ABA^{-1}B^{-1})$[1] in planning the input (a simple commutator is used at step 2, and a commutator of commutators at step 3). The final sequence of steps can however be written more compactly by imposing some further conditions, reducing the redundancy of

[1] the inverse A^{-1} of an input $A : [0, T_A] \to \mathbb{R}_m$ is defined here as $A^{-1} = -A(T_A - t)$

solutions to the equations in step 3 but not compromising generality:

A) If $v_f \neq v_0$, setting $\mu_2 = -\mu_1, \mu_4 = \mu_3$, and $\mu_5 = \mu_6$, a control sequence is obtained $(\mu_0+\mu_3, \mu_1+\mu_5, -\mu_3+\mu_7, -\mu_5, -\mu_7, \mu_5, \mu_3, -\mu_5, -\mu_3+\mu_7, \mu_5, -\mu_7, -\mu_5)$, that steers from $\mathbf{z}_0$ to $\mathbf{z}_f$ in just $12T$.

B) If otherwise $v_f = v_0$, the control sequence $(0, \nu_1, \nu_0 + \nu_2, -\nu_1 + \nu_3, -\nu_2, -\nu_3, \nu_2, \nu_1, -\nu_2, -\nu_1 + \nu_3, \nu_2, -\nu_3, -\nu_2)$, steers the system using 4 indeterminate variables.

4 Experimental

Introducing DxGrip–II. To experimentally validate the results of the theoretical work on manipulation by rolling conducted in the past years, the research group at Centro "E. Piaggio" of the University of Pisa designed and built two prototype end–effectors. The first "dextrous gripper" consisted of two parallel plates controlled by prismatic joints, and was described in [5]. The design of the second generation dexterous gripper (DxGrip–II) is described in fig.1, while a picture of a laboratory prototype is reported in fig.3. The gripper has two parallel jaws translating independently, and two turning disks with direct–drive motors on each jaw. Each finger is driven by a double four–linkage mechanism, which allows smooth transitions through singularities, and is endowed with a 6–axis force/torque sensor, which is used both for tactile detection of the contact point and for grasping force control. The combination of the two finger sensors is an effective alternative to a wrist–mounted force/torque sensor, which does not suffer from inertial effects of the end–effector mass. It should be pointed out that, while the use of turntables at the fingers of a gripper has already been proposed by Nagata [18], DxGrip–II has the possibility of translating the center of one turntable with respect to the other, thus achieving higher dexterity and most importantly the ability of rolling an object in all directions between the fingers. Like Nagata's hand, DxGrip–II is capable of turning a screwdriver and of reorienting parts on a conveyor belt without grasping them (using the exterior part of the turntables only, pressed against the surface of the object, and commanding their angular velocity suitably). In addition, our new hand can arbitrarily relocate and reorient any convex body with regular surface by rolling it among the fingers, thus justifying the name "dexterous gripper".

Surface reconstruction by rolling. The experimental results obtained by exploring an unknown object and reconstructing its shape using the techniques described in 2, are reported in figure fig.4. Figures show apparently how using few spherical harmonics (low N) and/or low regularization weights λ provides "bumpy" reconstructions, while heavy regularization tends to round up the object shape excessively. The correct tradeoff in filtering has to be decided on the basis of a working knowledge of the sensor noise statistics and of the application domain.

Planning a reconstructed surface. As an example of application of the planning algorithm described in section 3, consider the problem of rolling an object

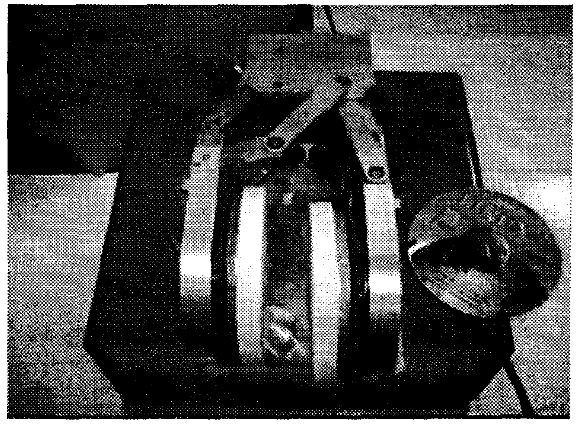

Figure 3: Prototype of the second generation dextrous end–effetor DxGrip–II.

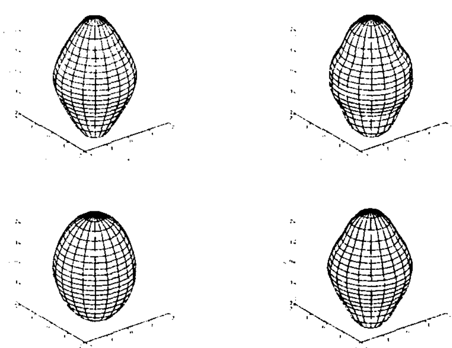

Figure 4: Exact description of the manipulated object (upper left), and approximations from experimental data with $\lambda = 0,002$ and $N = 7$ harmonics (underdamped, upper right), with $\lambda = 0,05$ and $N = 9$ (overdamped, lower left), and with $\lambda = 0,002$ and $N = 9$ (lower right).

whose reconstructed description is given, in terms of the spherical harmonics series (6), by $f_{0,0} = 1, f_{1,0} = 0.4, f_{2,0} = 0.1$ (see fig.5). The inputs resulting from application of the algorithm, modified as in Remark 1–A), to the problem of steering from $\mathbf{z}_0 = (-\pi/4, \pi/4, 0, 0, 0)$ to $\mathbf{z}_f = (\pi/4, -\pi/4, 0, 2, 1)$, are computed as $\mu_0 = \pi/2, \mu_1 = -\pi/2, \mu_3 = -0.40; \mu_5 = 3.03; \mu_7 = 1.07$. The solution of the system of nonlinear equations at the last step of the algorithm is performed numerically. The path followed by the coordinates along the 12 intervals used for planning are reported in fig.6.

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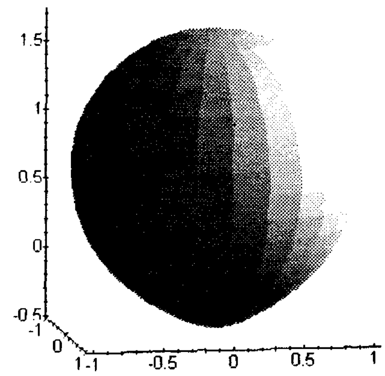

Figure 5: Shape of the convex object used in the example.

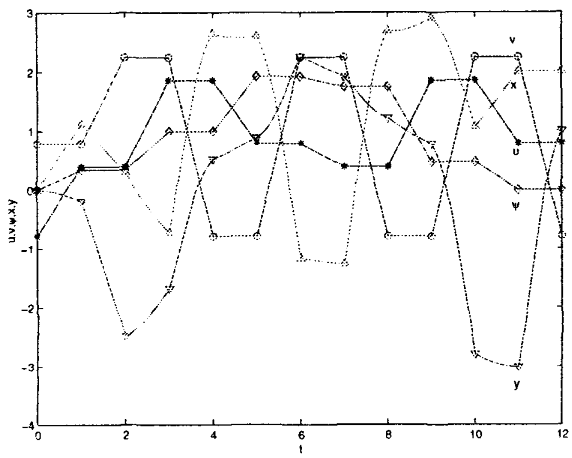

Figure 6: Plot of the state trajectories in the planning example.

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