Observability and Controllability Verification in Multi-Agent Systems through Decentralized Laplacian Spectrum Estimation

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***Abstract*— In this paper we show how the decentralized estimation of the spectrum of a network can be used to infer its controllability and observability properties. The proposed approach is applied to networked multi-agent systems whose local interaction rule is based on Laplacian feedback. We provide a decentralized necessary and sufficient condition for observability and controllability based on the estimated eigenvalues. Furthermore we show an example of application of the proposed method and show that the estimated spectrum can also be envisioned as a tool for decentralized formation identification.**

I. Introduction

Multi-agent systems composed by networks of unmanned mobile vehicles are envisioned to perform the most various tasks in the near future. The design of control algorithms for such systems poses several challenges to achieve robustness and scalability. So far such properties are expected to be achieved by decentralized control algorithms that make locally use of available information [1], [2], [3], [4].

A significant example of a multi-agent system is one involving agents with simple integrator dynamics under Laplacian feedback [2]. While the model of the agents' dynamics is clearly oversimplified, the network model has just the right complexity to capture several relevant features of a networked system linked to the topology of the network. Furthermore such model is widely accepted to be a good starting point in modeling leader-follower networks of mobile vehicles [5] with the aim of allowing a single pilot to control a multitude of mobile vehicles with limited available information.

Recently, an algorithm for the decentralized estimation of the eigenvalues of the Laplacian matrix associated to a network was proposed in [6]. Such algorithm consists in the agents applying a local state update rule that allows to infer the eigenvalues of the network through the application of a simple Discrete Fourier Transform to their state trajectory.

In this paper we build on the idea to use the information about the spectrum of the network to infer in a decentralized fashion properties such as controllability, observability and, more in general, its topology.

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The link between the network topology of a multi-agent system and its properties of controllability and observability have been deeply studied over the past few years (see for example [7], [8], [9], [10]). In [10] a graph theoretic sufficient and necessary condition for controllability has been developed. It turned out that controllability and, by duality, observability depend on the existence of external equitable partitions on the graph representing the network. The novelty in our contribution is that our condition is *locally checkable online*.

Another application of the information about the spectrum of a network is the identification of its topology. In general, the spectrum is not necessarily a unique identifier for a given topology. Moreover, in multi-agent systems we may be interested in the subproblem of estimating when a particular topology known a priori has been achieved. The target topology in which the agents are supposed to be in their nominal state of operations can be built so that it is identifiable by its spectrum. A strong application of this information is the enabling of a simple Luenberger observer to estimate correctly the relative position of each agent in the network with respect to the leader in absence of communication, GPS or common reference frames. In this paper we show how line and lattice formations composed by a convoy of n agents can be identified by their corresponding spectrum.

We point out that the theory presented in this paper can be easily extended to heterogeneous networks where a different weight is associated to each link.

This paper is structured as follows:

- In Section II we provide some background on the work in [6] regarding the decentralized estimation of the eigenvalues of a network topology.
- In Section III we present the main result of this paper, i.e. a decentralized method to check for observability and controllability.
- In Section IV we propose as research direction the use of the spectrum of a graph as a tool for formation identification.
- In Section V we present an application of the results presented in this paper.

II. Background

In multi-agents systems, it is common to let the nodes of a graph represent the agents, and to let the arcs in the graph represent the inter-agent communication links. In fact, this interaction graph plays a central role in representing the information flow among the agents, and in defining the properties of the system.

Let the undirected graph G be given by the pair $(V, \mathcal{E})$, where $V = \{1, \ldots, n\}$ is a set of n vertices, and $\mathcal{E}$ is a set of edges. Two nodes j and k are neighbors if $(j, k) \in \mathcal{E}$, and the set of the neighbors of the node j is defined as $N_j = \{k : (j, k) \in \mathcal{E}\}$. The degree of a node i, Δ_i is given by the number of its neighbors, we denote $\Delta_{max} = \max_i \Delta_i$. A graph G is *connected* if there is a path between any pair of distinct nodes, where a path $i_0 i_1 \ldots i_S$ is a finite sequence of nodes such that $i_{k-1} \in N_k$ with $k = 1, 3 \ldots S$.

In this paper we let the state of each node, x_i, be a scalar. (This does not affect the generality of the derived results.) The standard, consensus algorithm consists in each agent performing the following state update law

$$\dot{x}_i(t) = \sum_{j \in N_i} (x_j(t) - x_i(t)), \tag{1}$$

or equivalently $\dot{x}(t) = -\mathcal{L}x(t)$, where $x(t)$ is the vector with the states of all nodes at time t, and $\mathcal{L}$ is the graph Laplacian. $\mathcal{L}$ can be obtained as $\mathcal{I}\mathcal{I}^T$, where $\mathcal{I} \in \mathbb{R}^{n \times p}$, ($p$ being the number of edges), is the *incidence matrix* of the graph, defined as

$$[\mathcal{I}]_{kl} = \begin{cases} 1 & \text{if node } k \text{ is the head of the edge } l \\ -1 & \text{if node } k \text{ is the tail of the edge } l \\ 0 & \text{otherwise,} \end{cases}$$

given an arbitrary orientation of the edges.

Under some connectivity conditions, the consensus algorithm (1) is guaranteed to converge, i.e. $\lim_{t \to +\infty} x_i(t) = g$, $i \in \{1, \ldots, n\}$, where g is a constant depending on $\mathcal{L}$, and on the initial conditions $x_0 = x(0)$. (See for example [11], [12], [13].)

As in [14], [15], [16], we imagine that a subset of the agents have superior sensing, computation, or communication abilities. We thus partition the node set V into a leader set L of cardinality n_l, and a follower set F of cardinality n_f, so that $L \cap F = \emptyset$ and $L \cup F = V$.

Leaders differ in their state update law in that they can arbitrarily update their positions, while the followers execute the agreement procedure (1), and are therefore controlled by the leaders.

Under the assumption (without loss of generality) that the first n_f agents are followers, and the last $n_l = n - n_f$ are leaders, the introduction of leaders in the network induces a partition of the incidence matrix $\mathcal{I}$ as

$$\mathcal{I} = \begin{bmatrix} \mathcal{I}_f \\ \mathcal{I}_l \end{bmatrix},$$

where $\mathcal{I}_f \in \mathbb{R}^{n_f \times p}$, $\mathcal{I}_l \in \mathbb{R}^{n_l \times p}$, and the subscripts f and l denote respectively the affiliation with the leaders and followers set. As a result, the graph Laplacian $\mathcal{L}$ becomes

$$\mathcal{L} = \left[\begin{array}{c|c} \mathcal{L}_f & \mathcal{L}_{fl} \\ \hline \mathcal{L}_{fl}^T & \mathcal{L}_l \end{array} \right], \tag{2}$$

with $\mathcal{L}_f = \mathcal{I}_f \mathcal{I}_f^T \in \mathbb{R}^{n_f \times n_f}$, $\mathcal{L}_l = \mathcal{I}_l \mathcal{I}_l^T \in \mathbb{R}^{n_l \times n_l}$ and $\mathcal{L}_{fl} = \mathcal{I}_f \mathcal{I}_l^T \in \mathbb{R}^{n_f \times n_l}$.

The system we now consider is the controlled agreement dynamics, in which agents evolve through the Laplacian-based dynamics

$$\begin{cases} \dot{x} = -\mathcal{L}_f x - \mathcal{L}_{fl} x_l \\ \dot{x}_l = u \\ y = -\mathcal{L}_{fl}^T x - \mathcal{L}_l x_l \end{cases} \tag{3}$$

where x is the state vectors of the followers, and $u(t)$ denotes the exogenous control signal dictated by the leaders.

In the proposed approach all the agents (both followers and the leader) execute the algorithm for estimating the eigenvalues of $\mathcal{L}$. We now recall that the leader-followers network evolves according to

$$\begin{cases} \dot{x} = -\mathcal{L}_f x - \mathcal{L}_{fl} x_l \\ \dot{x}_l = u(t) \end{cases} \tag{4}$$

The leader has full access to the state of its neighbors and as such it is able to estimate

$$y = -\mathcal{L}_{fl}^T x - \mathcal{L}_l x_l \tag{5}$$

It follows that if the leader applies the following feedback control law,

$$u(t) = -\mathcal{L}_{fl}^T x - \mathcal{L}_l x_l + \hat{u}(t) \tag{6}$$

including the state of the leader with the others, the networked system can be described by

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{x}_l \end{bmatrix} = -\mathcal{L} \begin{bmatrix} x \\ x_l \end{bmatrix} + Bu \\ y = C \begin{bmatrix} x \\ x_l \end{bmatrix} \end{cases} \tag{7}$$

where $C = B^T = [0, \ldots, 0, 1]$.

A. Decentralized Laplacian Eigenvalues Estimation

In this section we review an algorithm recently proposed in [6] for the decentralized estimation of the Laplacian eigenvalues of a network.

The algorithm consists on having the network of agents simulate numerically the following dynamical system:

$$\begin{cases} \dot{z}_{1i}(t) = \sum_{j \in \mathcal{N}_i(t)} (z_{2i}(t) - z_{2j}(t)), \\ \dot{z}_{2i}(t) = -\sum_{j \in \mathcal{N}_i(t)} (z_{1i}(t) - z_{1j}(t)). \end{cases} \tag{8}$$

The above local interaction rule is meant to be simulated through the use of local communications between the agents, and thus no sensing of relative positions is involved in this step.

The behavior of the network state when each agent performs the above updating rule can be described by the following time varying autonomous linear system

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \mathcal{A}(t) \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \tag{9}$$

where

$$\mathcal{A}(t) = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathcal{L}(t) \\ -\mathcal{L}(t) & \mathbf{0}_{n \times n} \end{bmatrix} \tag{10}$$

and $\mathbf{0}_{n\times n}$ is the null $n \times n$ matrix.

This is a linear switching system, where the linear dynamics switch to a different one when an edge is added or removed from the network.

The resulting state trajectory is composed by a linear combination of sinusoids with frequencies corresponding to the Laplacian eigenvalues of the network. The following theorem characterizes the spectrum of the agents' trajectories which can be computed locally and independently by each single agent by applying the *Discrete Fourier Transform* (DFT) to a sufficiently long time window as showed in [6].

***Theorem** 1:*

Let us consider a system described by eq. (9) relative to a network whose graph $\mathcal{G}$ is *connected*. The module of the Fourier transform of the i-th state components $z_{1i}(t)$ and $z_{2i}(t)$, $i = \{1,\ldots,n\}$, can be written as

$$|\mathcal{F}[z_{1i}(t)]| = |Z_{1i}(f)| = a_{1,i}\,\delta(0) + \sum_{j=2}^{m} \frac{a_{j,i}}{2}\,\delta(f \pm \lambda_j/2\pi),$$

$$|\mathcal{F}[z_{2i}(t)]| = |Z_{2i}(f)| = b_{1,i}\,\delta(0) + \sum_{j=2}^{m} \frac{b_{j,i}}{2}\,\delta(f \pm \lambda_j/2\pi),$$

where $a_{j,i}$ and $b_{j,i}$ are appropriate real constants. ■

It is relevant to point out that the multiplicity of the eigenvalues can not be retrieved from the spectrum $Z_{1i}(f)$ or $Z_{2i}(f)$.

Clearly the amplitude of the frequency peaks in the resulting spectrum are function of the network eigenvectors and initial conditions [6]. Furthermore, in the case the network is not observable, some coefficients might be zero.

It is relevant to point out that such algorithm produces sustained oscillations at frequencies corresponding to the eigenvalues only if the initial conditions are not orthogonal to any eigenvector of the graph Laplacian.

Let C be any $m \times n$ matrix. The following result is stated for a general case and holds for any C, however in our application we consider the output matrix C as a $1 \times n$ matrix where all elements are zeros but for the i-th element that is equal to 1 where i is the ID of the agent observing the network. Let $O(\mathcal{A},\hat{C})$ and $O(\mathcal{L},C)$ be the observability matrices of system (9) and system (7) .

The following result, which has been proved in [6], states that the observability properties of system (9) are strictly related to the observability properties of the multi-agent network under Laplacian feedback described by system (7).

***Theorem** 2:*

Let $\mathcal{A}$ be the matrix describing the group dynamics as in (10). Let C be any $m \times n$ matrix. The following properties hold:

- $\text{rank}(O(\mathcal{A},\hat{C})) = 2\text{rank}(O(\mathcal{L},C))$.
- $(\mathcal{A},\hat{C})$ is observable if and only if $(\mathcal{L},C)$ is observable.

Proof: See [6]

□

We now state the procedure implemented by each agent to estimate the eigenvalues of the network topology.

Algorithm 1: Eigenvalue Estimation Algorithm

Data: Each agent stores in its memory two variables z_1, z_2 to numerically simulate the proposed interaction rule (9). A sampling time τ is chosen by the numerical method chosen to simulate the interaction rule.

Result: $\lambda(\mathcal{L}) = \{\lambda_1,\lambda_2,\ldots,\lambda_m\}$.

Estimation steps:

1) Each agent $i = 1,\ldots,n$ numerically simulates $\dot{x} = \sum_{j\in\mathcal{N}_i}(x_j - x_i)$ with random initial conditions.
2) At any instant of time, agent i computes the Discrete Time Fourier Transform of $x_i(t)$ for the time window $[t, t-t_0]$.
3) Agent i computes the location of the peaks (spectral lines) in the computed DFT.
4) The location of the peaks corresponds to the $m \le n$ eigenvalues corresponding to the observable modes and are given as output.

***Remark** 1 (Implementation Remarks):* Algorithm 1 is based on the numerical simulation of system (9). For this reason the eigenvalues estimation procedure requires to compute the discrete time Fourier Transform over some sufficiently long time window to minimize approximation errors.

Furthermore to correctly observe the network eigenvalues we need to choose the number of samples m of the time window and the sampling frequency ω_s. Regarding the sampling frequency it has to be at least twice as the maximum frequency contained in the signal, which if no topology switching occurs during such time window corresponds exactly to λ_{max}. Since it holds $\lambda_{max} \le \Delta_{max} \le n-1$ it is sufficient to impose $\omega_s \ge 2\Delta_{max}$ or $\omega_s \ge 2(n-1)$.

$$\begin{cases} \omega_s \ge 2\lambda_{max} \\ \lambda_{max} \le \Delta_{max} \le n-1 \end{cases}$$

■

III. Decentralized Check for Observability and Controllability

In this section we present a method for the decentralized online verification of observability and controllability in a multi-agent system. In the following it is assumed that the agents execute Algorithm 1 and thus each agent estimates the eigenvalues (without multiplicity) observable from its position by taking only its own state trajectory as output. The basic idea is to exploit the properties of algorithm (9) to locally estimate the spectrum of the network and then link this information to check for observability and controllability. Such link is made possible by the fact that the modes of system (9) are observable if and only if the modes of system (7) are observable.

We now provide some basic helpful facts of linear system theory.

***Lemma** 3:* System (4) is controllable if and only if system (7) is controllable.

Proof:

System (4) differs from system (7) in that the leader applies the following feedback control law

$$u(t) = -\mathcal{L}_{fl}^T x - \mathcal{L}_l x_l + \hat{u}(t),$$

where $\hat{u}(t)$ is an input with the same dimensions as $u(t)$. If the system is controllable with such feedback it is controllable also with $u(t) = \hat{u}(t)$ since the input enters only in the row corresponding to x_l. Necessity comes from the fact that if system (7) is not controllable from $\hat{u}(t)$ then it is not controllable from any input entering in the row of x_l and thus also

$$\bar{u}(t) = \hat{u}(t) + \mathcal{L}_{fl}^T x + \mathcal{L}_l x_l = u(t),$$

proving the statement. ■

***Lemma** 4:* If the Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ has eigenvalues with multiplicity greater than one, then system (7) is not observable/controllable.

Proof:

See [17] chapter 9.5. ■

Now we state one of the main results of the paper. In the following theorem a sufficient and necessary condition for observability and controllability verification is given. Such condition involves only the local information available to agent i if the total number of agents n is known.

***Theorem** 5:* Let the network of agents be represented by a connected graph $\mathcal{G}$. Assume each agent estimates the eigenvalues of system (9), by applying the *DFT* algorithm to its state trajectory. Let agent i know the total number of agents n connected to the network. Then the network described by

$$\begin{cases} \dot{x} = -\mathcal{L}_f x + \mathcal{L}_{fl} u \\ y = \mathcal{L}_{fl}^T x \end{cases} \tag{11}$$

is observable and controllable from agent i if and only if agent i observes n distinct eigenvalues.

Proof:

- Sufficiency:

Assume agent i observes n modes of system (9) and they are distinct, then by taking as output the matrix $C = [0, \ldots, 1, 0, \ldots]$ with 1 in the i-th element, we have that observability matrix $(C, \mathcal{L})$ is full rank due to theorem 2. Due to lemma 3 if system (7) is controllable so is system (11). Furthermore since system (11) is symmetric and $C = B^T$, by duality the system is also controllable.

- Necessity:

Assume agent i estimates n distinct eigenvalues, assume system (9) is initialized with an initial condition not orthogonal to any of its eigenvector. If system (11) is not observable, then the observability matrix $(C, \mathcal{L})$ must be rank deficient and so has to be the observability matrix for system (9). It follows that if system (9) is not observable, then by definition the number of observable modes must be less than n which is a contradiction. Furthermore observability of system (7) is a necessary condition for the observability of system (11), the same goes for controllability. ■

The above theorem allows the agents to estimate in a decentralized fashion some relevant properties of the network if the number of agents is known. Note that the necessary condition holds only if system (9) is initialized with a proper initial condition so that all the system modes are excited. Now suppose that the total number of agents is not known and that the actual network is eventually not controllable nor observable. We are interested in finding the dimension of the controllable/observable subspace from any given agent. The following theorem characterizes the dimension of the controllable/observable subspace as function of the number of observable eigenvalues of system 9 which is simulated for the execution of Algorithm 1.

***Theorem** 6:* Assume each agent estimates the eigenvalues of system (9), by applying the *DFT* to its state trajectory. Assume agent i estimates a number of distinct eigenvalues m_i.

The dimension of the controllability/observability subspace from agent i is equal to m_i.

Proof:

Assume agent i observes m_i eigenvalues executing Algorithm 1. Thanks to theorem 2 we have that

$$\text{rank}(O(\mathcal{A}, \hat{C})) = 2\text{rank}(O(\mathcal{L}, C)).$$

Since the eigenvalues of system $\mathcal{A}$ are purely imaginary, pairwise conjugate and equal to the eigenvalues of $\mathcal{L}$ in modulus, we have

$$\text{rank}(O(\mathcal{L}, C)) = m_i.$$

■

***Remark** 2:* Theorem 6 holds if system (9) is initialized with a proper initial condition so that each system mode is excited. In the case such condition cannot be guaranteed, then the dimension of the controllability/observability subspace from agent i is clearly greater than or equal to m_i. ■

IV. Spectrum based Formation Identification

The idea of estimating topological features of a graph from its spectrum has been around for quite some time in algebraic graph theory. Unfortunately it has been shown that the spectrum of a graph is not a unique identifier for its topology. As an example, if two graphs are identical except for a relabeling of their nodes then necessarily the two spectra are identical. On the other hand there exists several graphs which are co-spectral with many others [18], [19], [20]. In this section we focus on the practical uses of this notion for the identification of regular structures such as formations of multi-agent systems.

A vast literature that deals with achieving some desired formation, e.g. [3], [21], [22], [23], [24], in a multi-agent system possibly in a decentralized fashion exists. A relevant issue in such decentralized approaches is to understand when such formation has been actually achieved so that the agents can switch mode of operation to something else.

It is clear that if the achievement of a formation could be linked directly to the spectrum of its topology then the numerical simulation of system 9 by the network and the execution of Algorithm 1 could provide an instance of solution to such problem.

A relevant class of graph topologies that serve our cause are those structured graphs whose eigenvalues are known analytically as function of the number of nodes.

The first of such graphs is the line graph, or path P_n of n agents whose eigenvalues are

$$\lambda(P_n) = 4\sin(\frac{\pi i}{2n})^2, \quad \forall i = 0, \ldots, n-1. \qquad (12)$$

This fact is relevant to practical applications in that the line graph is both controllable and observable for leader-follower networks. Furthermore it has obvious applications in the control of convoys of ground vehicles.

Since the cartesian product of graphs has eigenvalues equal to any combination of summation of the eigenvalues of the original graphs [25], we have that the $n \times m$ grid has eigenvalues given by

$$\lambda(G_{n\times m}) = 4\sin(\frac{\pi i}{2n})^2 + 4\sin(\frac{\pi j}{2n})^2, \quad \forall i, j = 0, \ldots, n-1.$$

The grid graph has significant applications in the coverage problem for both multi-agents systems and sensor networks.

V. Application to Leader-Follower Networks

In this section we apply the proposed method for decentralized observability verification to a leader-follower network and present an example of spectrum-based formation identification.

Let us consider a group vehicles with the task to form a convoy and move toward a target. Suppose that the leader knows the number of agents of the network and the desired topology which is determined by the eigenvalues of the Laplacian Matrix. Furthermore, suppose that each agent is provided with a decentralized controller which is able to chose its neighbors in order to reach the desired topology.

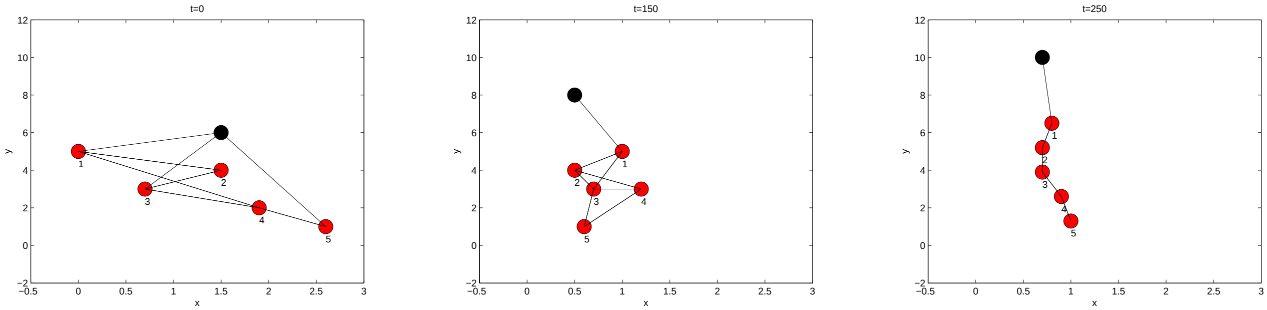
Starting from the initial point and structure of Figure 1(a), the communication links among nodes are changing (Figure 1(b)) to the final structure of Figure 1(c).

Figure 2 shows the evolution of the eigenvalues of the Laplacian matrix associated with networks of Figure 1(a), 1(b), 1(c). For every t it reports the Discrete Fourier Transform (DFT) to a sufficiently long time window (of size T_w) of the trajectory of the state of the system (9) in the interval $[t - T_w, t]$ which is composed by a linear combination of sinusoids with frequencies corresponding to the Laplacian eigenvalues of the network. It is clear that, since the window is sliding, we are able to capture the eigenvalues of the Laplacian matrix associated to the network Figure 1(c) $\forall t \in [0, T_f]$ with all their modification to the final set-up. In particular, at time $t = 0$ we can see from Figure 2 that the topology in Figure 1(a) is not controllable and observable from the leader since it has eigenvalues located in $\lambda(\mathcal{G}_1) = [0, 1.4, 3, 3, 3, 5.5]$, only 4 distinct eigenvalues with 6 agents. At time $t = 150$ the topology in Figure 1(b) is completely controllable and observable from the leader since we observe 6 eigenvalues on 6 agents. Since the desired formation is a line and its eigenvalues are known, we can infer that at time $t = 150$ the agents are not in a line-graph since its spectrum is $\lambda(\mathcal{G}_2) = [0, 0.7, 2.1, 3.4, 4.5, 5.1]$. At last, at time $t = 250$ Figure 2 shows that the network in Figure 1(c) is still controllable and observable and the observed spectrum matches the one of a line graph $\lambda(\mathcal{G}_3) = [0, 0.2, 1, 2, 3, 3.7]$ according to (12).

It is clear that this context emphasize the importance of the proposed method: by executing the decentralized check all agents are able to investigate about the eigenvalues of the network 2 and to settle whether the network is changed and whether the actual configuration is the desired one, for example observable. Only in the latter case, the leader, from which the network is completely observable, is interested in reconstruct the connection scheme through which it is able to know all information regarding the other node in the network.

VI. Conclusion

In this paper we proposed a decentralized method for online checking of controllability and observability of a network of single integrators with Laplacian feedback. The method exploits the knowledge of the eigenvalues of the linear dynamics made available by a recently proposed algorithm in [6]. We proposed the use of the spectrum of the network of a multi-agent system to identify when a desired formation has been achieved. Finally we presented an application in which the proposed method is used to check for controllability and observability of a convoy of vehicles and shown that the convoy, whose topology corresponds to a line-graph, can be identified in a decentralized way from the Laplacian spectrum of the network.



(a) (b) (c)

Fig. 1. The initial structure of the convoy (a) and its modifications (b) toward the desired topology (c).

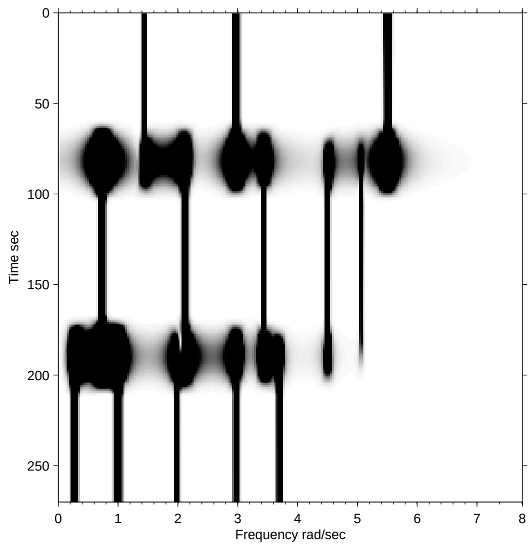


Fig. 2. Spectrogram of the switching topology in figure 1(a),1(b),1(c) obtained by algorithm 9.

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