A Decoupled Impedance Observer for a Variable Stiffness Robot

Alessandro Serio[*,+], Giorgio Grioli[*], Irene Sardellitti[+], Nikos G. Tsagarakis[+] & Antonio Bicchi[*,+]

***Abstract*— This paper focuses on the estimation of the impedance for a Variable Impedance Actuator (VIA) through torque and position measurements. Despite the recent development of several VIA, impedance control is not yet implemented in closed loop because of the difficulty of obtaining in real-time measurements of time-varying impedance. The estimation algorithm is proposed as an alternative approach to the standard procedures of impedance identification, to robustly tolerate the variability of the mechanical stiffness due, for example, to model uncertainties. The impedance estimator is therefore implemented on the Actuator with Adjustable Stiffness (AwAS). The effectiveness of the proposed estimator is proved through simulation and experimental results.**

I. INTRODUCTION

While most of today's robots are built with rigid links and joints, recent robotic research shifted toward a new paradigm of intrinsically compliant robots. The first solutions of this kind introduced simple linear springs between the actuators and the links of a robot [1]. This approach was improved by realizing actuators with integrated adjustable stiffness, where springs, which could be tuned to the particular task in early prototypes [2], and could be adjusted in real-time on more recent devices ([3], [4] and [5]). Recently, devices which can also regulate damping or inertia have been proposed [6], thus generalizing Variable Stiffness Actuators (VSA) in Variable Impedance Actuators (VIA)

The overall trend aims toward the development of VIA [7] that can adapt to the particular tasks and even during the task itself, changing the shape of their output dynamic characteristic, possibly with more than one degree of freedom.

Development of such novel actuators gives rise to interesting problems in term of control. A number of recent papers tackle the problem of controlling variable stiffness devices. Approaches, ranging from simple PD control [8] to more sophisticate feedback linearization techniques [9], are adopted to control stiffness in VIAs. Nevertheless, most of these approaches suffer from the same flaw: the impedance is not obtained trough direct measurements but it is inferred from the mathematical model of the device. Even if modeling an actuator in order to derive its stiffness seems like an easy and feasible approach, three main obstacles render it problematic. First of all, the derivation of a model requires knowledge of the non-linear elastic mechanism of the actuator, and while this is possible today on prototypical devices, could not be facilitated or even hindered in tomorrow's commercial devices. Second, derivation of impedance from a model requires fine calibration of the model parameters, in fact, due to intrinsic non-linearities of most VIAs, small errors in the model can badly propagate and produce large errors in the reconstruction of impedance. Third, a model approach does not easily account for parameter variation due to wear, change in external condition (e.g. temperature) and unmodeled dynamics.

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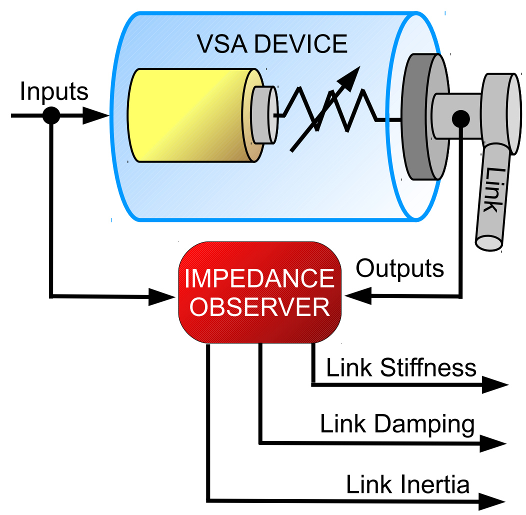


Fig. 1. Concept of impedance observer for a variable stiffness robot: inertia, damping and stiffness on the link side are estimated from measures of inputs (motors currents) and outputs (angles and torques) of VSA device.

In a recent work [2], a stiffness observer was proposed as an alternative approach to face this problem. Without relying on a detailed model of the actuator, but rather using measurements of forces, positions and their derivatives, the method is able to reconstruct the time-varying value of stiffness. Its application to the case of VSA-powered robots is partially restricted by the necessity of knowing estimates of damping and impedance of the link, although the algorithm shows some robustness to small errors on this information.

When linear impedance parameters are sufficient to model a system, standard estimation techniques exist to solve this problem, such as the Extended Kalman Filtering (EKF). Given proper calibration, their performance is satisfactory. A combined estimation approach is proposed to jointly observe non-linear stiffness and linear damping and inertia parameters. However, the first proposed implementation is not always practical: due to the fact that a loop between the two observers arises, robustness of the observation stability is severely undermined.

Therefore we propose a technique for the combined estimation of the whole set of impedance parameters of a VIA powered system (as in Fig. 1), which avoids interacting observation loops, thus preserving robustness of the estimation. The derived method can be applied to a class of variable stiffness devices. Its practical feasibility is demonstrated by applying it to the estimation of the impedance parameters of the AwAS variable stiffness actuator, in simulations first and experimentally as a final verification.

This document is organized as follows: in section II we state the general frame of the problem, section III presents our solutions, section IV shows result obtained both in simulations and experiments. Conclusions are drawn in section V.

II. PROBLEM STATEMENT

Most of the VIA mechanisms are characterized by 3 degrees of freedom (DOFs) and, therefore, can be described by a similar common structure. We will assume here that a VSA can be modeled with one motor (indirectly) actuating the link movement and another motor actuating the impedance variation, as in Fig. 1, or:

$$\begin{cases} I\ddot{q} + N\dot{q} + \Sigma(\theta_2,\cdots) = \tau_{ext} \\ B_1\ddot{\theta}_1 + D_1\dot{\theta}_1 - \Sigma(\theta_2,\cdots) = \tau_1 \\ B_2\ddot{\theta}_2 + D_2\dot{\theta}_2 + \Gamma(\theta_2,\cdots) = \tau_2 . \end{cases} \quad (1)$$

The first equation of (1) represents the dynamics of the link: I, N and $\Sigma(\theta_2,\cdots)$ are inertia, damping and the non-linear variable impedance of the link, respectively, τ_{ext} is the external torque on the link, q is the link angle. The second equation of (1) represents the dynamics of the position actuating motor: B_1 and D_1 are inertia and damping of the link motor, respectively, τ_1 is the motor torque andθ_1 is the motor angle. The third equation represents the dynamics of the impedance actuating motor: B_2 and D_2 are inertia and damping of the stiffness motor, respectively $\Gamma(\theta_2,\cdots)$ is the torque needed for the motors to change impedance and τ_2 and θ_2 are the motor torque and angle, respectively.

The functions

$$\Sigma(\theta_2,\cdots) = \Sigma(\theta_2, q-\theta_1, \dot{q}-\dot{\theta}_1,\cdots) \quad (2)$$
$$\Gamma(\theta_2,\cdots) = \Gamma(\theta_2, q-\theta_1, \dot{q}-\dot{\theta}_1,\cdots), \quad (3)$$

represent the *variable impedance* part of the system. Their effective structure and the set of their arguments itself depend on the particular VIA system considered, but it can usually be restricted to the values of θ_2 , $q-\theta_1$ and their derivatives. In the case of a VSA, i.e. a VIA where damping and inertia are constant, the arguments of $\Sigma()$ and $\Gamma()$ usually reduce to just θ_2 , $q-\theta_1$, respectively.

A. Variable Stiffness Estimator

In [2] an approach for the measurement of the stiffness component of variable impedance actuators was proposed, which is shortly resumed here for convenience. Assuming that the stiffness-regulating input $\theta_2(t)$ and its first derivative $\dot{\theta}_2(t)$ are bounded, more precisely assuming that the ratio between the stiffness regulation rate of change and the velocity of the measured trajectory is bounded, namely that, for all times t during the application of the observer, it holds

$$\frac{|\dot{\theta}_2(t)|}{|\dot{q}(t)|} < v \in \mathbb{R}, \quad \forall t.$$

By differentiating the link dynamics in (1) with respect to time, yields

$$\dot{\tau}_{ext} = I\dddot{q} + N\ddot{q} + K(\dot{q}-\dot{\theta}_1) + \Sigma_u\dot{\theta}_2, \quad (4)$$

where $K = \frac{\partial\Sigma}{\partial\delta}$, with $\delta = q-\theta_1$, is the link-side stiffness of the VIA device [1], $\Sigma_u = \frac{\partial\Sigma}{\partial\theta_2}$ represents the effect of θ_2 on the force exerted by the motor on the link. If an estimate $\hat{K}(t)$ of the stiffness is available, it can be used to build a best-effort prediction for $\dot{\tau}_{ext}$ as

$$\dot{\hat{\tau}}_{ext} = I\dddot{q} + N\ddot{q} + \hat{K}(\dot{q}-\dot{\theta}_1) . \quad (5)$$

Let $\tilde{K}(t) = K - \hat{K}(t)$ be the estimation error. The update law

$$\dot{\hat{K}} = \alpha\dot{\tilde{\tau}}_{ext}\mathrm{sgn}(\dot{q}-\dot{\theta}_1), \quad (6)$$

with $\alpha > 0$ and $sgn(x)$ defined as usual, is shown to be such that the estimation error is uniformly ultimately bounded, with the bound given by

$$|\tilde{K}| > \frac{|K_q|}{\alpha} + \left(|s_u| + \frac{|K_{\theta_2}|}{\alpha}\right) v . \quad (7)$$

K_q and K_{θ_2} are partial derivatives of K with respect to q and θ_2 respectively.

This approach has the disadvantage of necessitating to knowledge of I and N, which may not always be available. A straightforward application of the former observer is, therefore, not possible if those parameters are ignored.

Standard methods to identify I and N rely on constant and known stiffness values. Moreover, an on-line estimation of I and N allows to address more general variable impedance

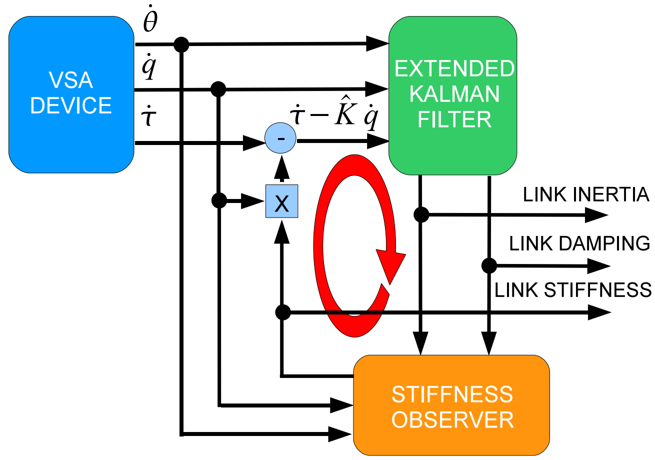


Fig. 2. Schematic of the observer as proposed in section III-A. The interaction loop between the two observers is highlighted.

[1]The total impedance on the link side depends on the behavior of θ_1 and θ_2 which, in turn, depends on the particular control scheme adopted.

actuators, although, at this stage we can only prove convergence of K for slowly varying I and N.

III. Impedance Observers

A. Combined EKF-Stiffness Observer

Assuming that a measurement of the torque τ_{ext} is known, a possible approach to the combined estimation problem relies on the juxtaposition of a stiffness observer and an EKF. Rewriting (4)as

$$\dot{\tau}_{ext} - K(\dot{q} - \dot{\theta}_1) - \Sigma_u \dot{\theta}_2 = \dot{\tau}_* = I\dddot{q} + N\ddot{q} \,, \quad (8)$$

an EKF can be easily built in to estimate the impedance parameters of the rightmost side given a measurement of $\dot{\tau}_*$. Given the estimates $\hat{I}$ and $\hat{N}$ derived from the EKF, the estimation of stiffness can be obtained by using the best-effort prediction for the $\dot{\tau}_{ext}$ defined now as

$$\dot{\hat{\tau}}_{ext} = \hat{I}\dddot{q} + \hat{N}\ddot{q} + \hat{K}(\dot{q} - \dot{\theta}_1) \,, \quad (9)$$

where $\hat{\tau}_{ext}$, $\hat{I}$, $\hat{N}$ and $\hat{K}$ are the estimations of external torque, inertia, damping and stiffness, respectively.

By virtue of the robustness of the stiffness observer algorithm claimed in [2], the error on the knowledge of I and N should introduce only an error on the estimate K.

The knowledge of $\dot{\tau}_*$ is, unfortunately, unavailable but, possesing possessing an estimate of the stiffness K, can be approximate as

$$\dot{\hat{\tau}}_* = \dot{\tau}_{ext} - \hat{K}(\dot{q} - \dot{\theta}_1).$$

This approach has the advantage of estimating the whole set of parameters using only torque and position measurement (and their derivatives) without needing any other assumption. It works under the hypothesis that the initial error on the estimates of I and N and the influence of the term Σ_u are small enough. Nevertheless, as it is highlighted from the block-diagram of Fig. 2, the problem of an interaction loop between the two observers arises. This has a negative effect on the stability of the algorithm, and can make convergence depend strongly on initial guesses. A solution to this problem is presented in the next section (III-B).

B. Decoupled Impedance observer

Assume that the torque sensor necessary for the stiffness estimation is assembled between the actuator unit and the link so as to measure Σ. It is possible to notice, that the variable impedance term Σ, giving rise to the stiffness K, appears in both the first and second equations of (1). If we consider, in particular, the second equation of (1), its general form is identical to that needed by the stiffness observer. While accomplishing the stiffness estimation task on the first equation of (1) requires the knowledge of I and N, performing the estimate on the second of (1) demands just the knowledge of the motor parameters B_1 and D_1. Those values can be usually deduced by the motor data-sheets, or otherwise measured with standard off-line calibration techniques [2]. The rest of the problem, i.e. the estimation of the inertia I and the damping N, can be realized with a standard EKF on the system

$$I\ddot{q} + N\dot{q} = \hat{u}. \quad (10)$$

Defining the extended state vector

$$\begin{bmatrix} q \\ \dot{q} \\ 1/I \\ N/I \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad (11)$$

allows to write the non linear discrete state representation of (10) as

$$\begin{cases} x_1^{(k+1)} = x_2^{(k)} T_c + x_1^{(k)} \\ x_2^{(k+1)} = (-x_3^{(k)} x_2^{(k)} - x_4^{(k)} \hat{u}^{(k)}) T_c + x_2^{(k)} \\ x_3^{(k+1)} = x_3^{(k)} \\ x_4^{(k+1)} = x_4^{(k)} \end{cases} , \quad (12)$$

where T_c is the sampling time. From (12), a suitable EKF can be designed which is effectively decoupled from the stiffness observer (for some details see the appendix or [10]). The stiffness observer, built in as explained in section II, is discretized such as

$$\hat{K}^{(k+1)} = [\alpha \dot{\hat{\Sigma}} \mathrm{sgn}(q^D - \theta_1^D)] T_c + \hat{K}^{(k)}, \quad (13)$$

with $\dot{\hat{\Sigma}}$ defined as

$$\dot{\hat{\Sigma}} \triangleq \Sigma^D - \hat{K}^D (q^D - \theta_1^D) - B_1 q^{DD} - D_1 q^{DDD}, \quad (14)$$

with x^D calculated, for a generic quantity x, as

$$[x^D]^{(k)} = \frac{x^{(k)} - x^{(k-1)}}{T_c}. \quad (15)$$

IV. Results

The impedance observer was tested through simulations and experiments on the Actuator with Adjustable Stiffness (AwAS), developed by the Italian Institute of Technology [11] (see Fig. 3). The dynamics of the AwAS actuator, neglecting the gravity, can be described by the following equations:

$$\begin{cases} I\ddot{q} + N\dot{q} + \tau_E = \tau_{ext} \\ B_1 \ddot{\theta}_1 + D_1 \dot{\theta}_1 - \tau_E = \tau_1 \\ B_2 \ddot{\theta}_2 + D_2 \dot{\theta}_2 + \tau_r = \tau_2 \end{cases} \quad (16)$$

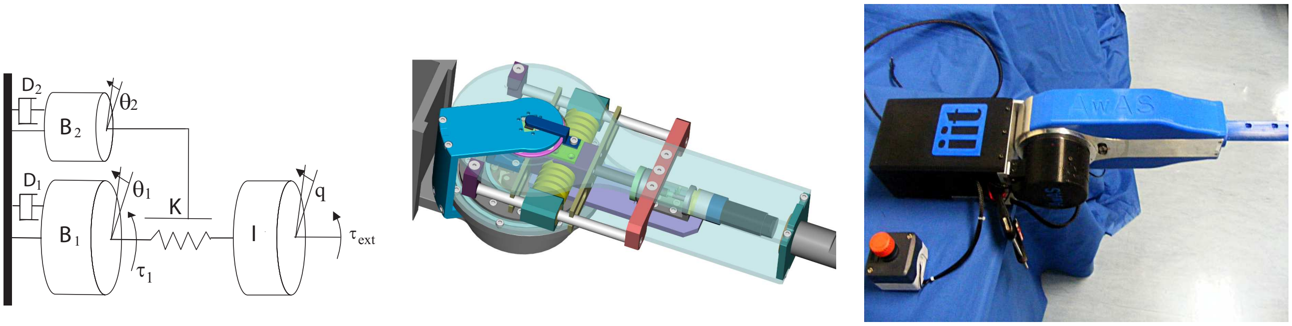
where I, N and M are the inertia, damping and mass of the link with generalized coordinate q; B_i, D_i and τ_i with $i \in [1, 2]$ are the inertia damping and command torque of the motors M_1 and M_2, respectively, with generalized coordinate θ_i. The external torque applied at the joint is represented with τ_{ext} and the elastic torque τ_E is formulated such as

$$\tau_E = k_s r^2 \sin(2\theta_s) \quad (17)$$

where k_s is the spring rate and $\theta_s = q - \theta_1$ is the spring deflection; the rotational stiffness $K = \frac{\partial \tau_E}{\partial \theta_s}$ is therefore obtained such as

$$K = 2k_s r^2 cos(2\theta_s). \quad (18)$$

[2]Moreover, small errors in the knowledge of these two parameters are robustly tolerated, as shown in [2].



(a) AwAS schematic (b) AwAS CAD (c) AwAS real prototype

Fig. 3. The Actuator with Adjustable Stiffness (AwAS) used as a testbed for the proposed impedance observer. Schematic (a), CAD image (b) and prototype (c).

The joint stiffness K depends on the lever arm r, which is the effective distance between the center of rotation of the joint and the springs, and, in minor contribution, from the deflection of the springs. The lever arm is adjusted through a ball screw mechanism through the actuator M_2 such as

$$r = r_0 - n\theta_2 \tag{19}$$

where n is the transmission ratio between the motor and the ballscrew and r_0 is the initial length. Finally, the torque τ_r which applies at the motor M_2 is given by

$$\tau_r = -2k_s nrsin(\theta_s)^2. \tag{20}$$

Note that, to simplify the notation the motors inertia and damping factors are already scaled by the transmission ratios.

A. Tuning

The observer was calibrated by trial and error as following.

1) Extended Kalman Filter: starting from Q, R e $P_{0|0}$ (see section VII) matrices equal to the identity, diagonal elements related to badly converging variables are tuned. [3]
2) Stiffness observer: the only parameter to calibrate is the observer gain α, it is obtained by optimizing the trade-off between the effects of the measurement noise on one side, and speed of convergence of the estimate on the other.

B. Simulations

Results of simulations are shown in Fig. 4. The simulated experiment consisted in feeding the two motors of the AwAS actuator with two sinusoidal torque signals. Parameters of the two observers are tuned as follows: the EKF (see section VII) matrices are

$$Q = 0.001 \times I_{4\times4}$$
$$R = 0.001 \times I_{2\times2},$$

with initial guesses

$$x_{0|0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$
$$P_{0|0} = 10000 \times I_{4\times4}.$$

The stiffness observer gain (6) is set to $\alpha = 2000$.

[3] In particular, elements of Q are related to oscillation of the variables, elements of R (see section VII) to the convergence speed. $P_{0|0}$, influences the update speed on the initial moments in which EKF starts.

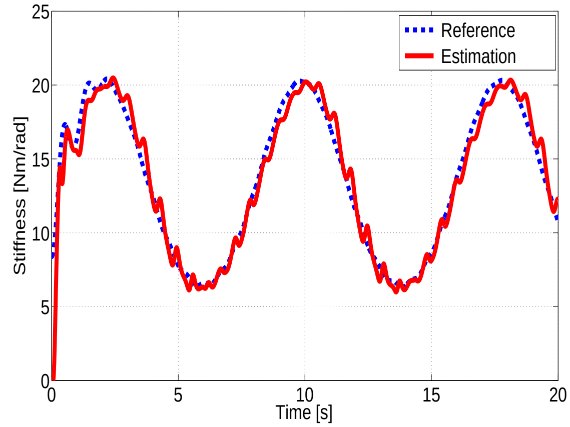
C. Experimental Setup

The setup of the AwAS system, employed for the execution of the experimental trials, is shown in Fig. 3. The AwAS unit consists of two actuators. The main joint actuator (Link Motor) is based on a combination of an Emoteq HT-2300 frameless brushless motor (capable of a peak torque of 2.3Nm) and a harmonic reduction drive CSD 20 (reduction ratio of $N = 50$ and peak rated torque of 80Nm). The stiffness adjusting actuator (Stiffness Motor) is realized by a DC motor from Faulhaber (peak torque of 0.8Nm) combined with a ball screw reduction drive which converts the rotary motion of this motor into a linear displacement, allowing to change the effective lever arm and efficiently tune the joint stiffness. More details on the mechanical implementation of the AwAS unit can be found in [11] and [12]. The sensing system of AwAS includes four position sensors and one torque sensor; one optical encoder measures the position of the link motor, two absolute magnetic encoders measure position of the joint before (at the harmonic drive output) and after the compliance module (link position) while an incremental encoder monitors the position of stiffness motor and subsequently the displacement of the linear drive. A torque sensor is located between the harmonic drive and the intermediate link and senses the torque applied by the link motor. The general specifications of AwAS are presented in Table I. The unit controller and power driver used to control the AwAS unit are custom control boards based on the Motorola DSP 56F8000 chip with CAN communication interface.

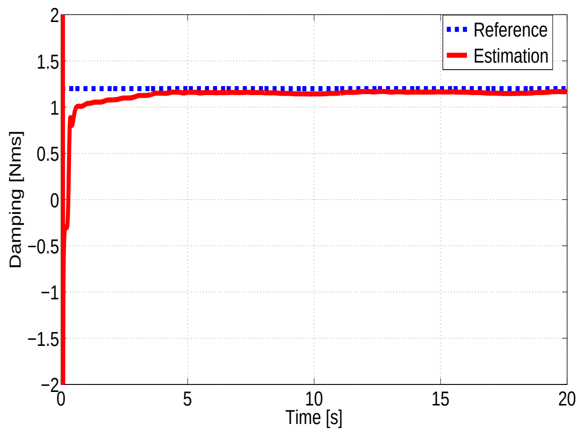
TABLE I
GENERAL SPECIFICATION OF AWAS

Range of Motion(deg)	-120÷120
Range of Stiffness (N m/rad)	30÷130
Peak Output Torque (N)	80
Length (m)	0.27
Width (m)	0.13
Total Weight (Kg)	1.8

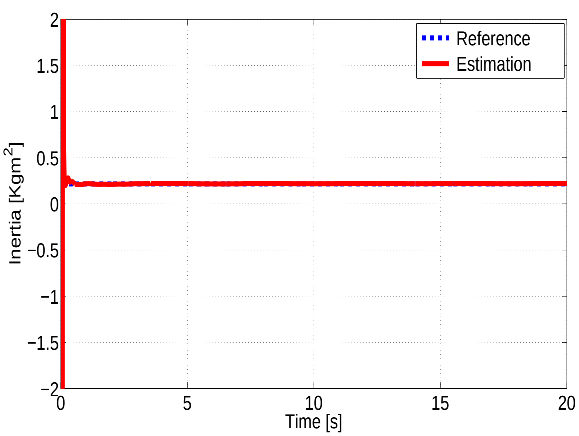
The experiment consisted in feeding the two motors of the AwAS actuator with two sinusoidal torque signals. Parame-



(a) Stiffness estimation

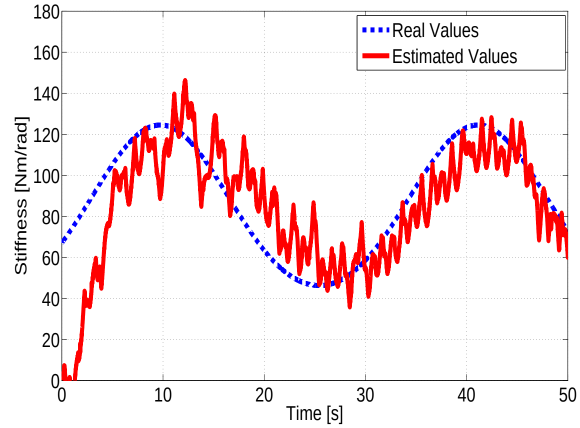


(b) Damping estimation

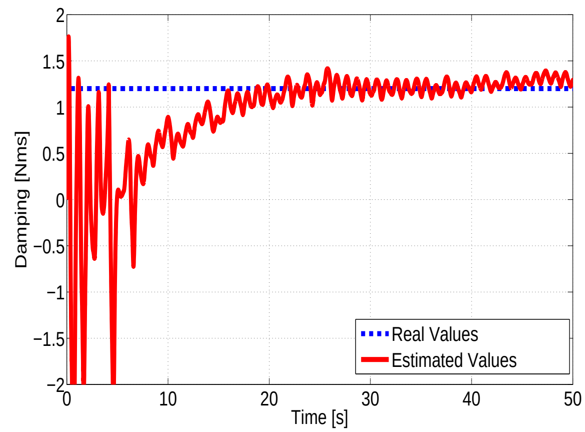


(c) Inertia estimation

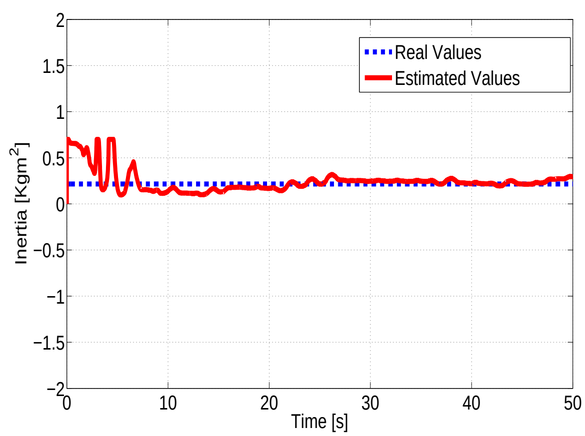
Fig. 4. Simulation result. Mean values of relative errors: 5.1% for the link stiffness, 6.7% for link damping and 9.8% for link inertia.



(a) Stiffness estimation



(b) Damping estimation



(c) Inertia estimation

Fig. 5. Experiment result. Mean values of relative errors: 38.2% for the link stiffness, 9.4% for link damping and 12.2% for link inertia.

ters of the two observers are tuned as follows: the EKF (see section VII) matrices are

$$Q = 0.00001 \times I_{4\times 4}$$
$$R = 0.000001 \times I_{2\times 2},$$

with initial guesses

$$\begin{aligned} x_{0|0} &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \\ P_{0|0} &= 100 \times I_{4\times 4}, \end{aligned}$$

while the stiffness observer gain (6) is set to $\alpha = 8$.

D. Results Discussion

Results of simulations are shown in Fig. 4, while results of experiments are presented in Fig. 5. The main differences that can be noticed consist in a solwer convergence speed and a larger error on the stiffness estimate. They arise mostly due to the quantization error on the position sensor, which has a negative relapse on the calculation of the derivatives, and force the gains to be smaller.

V. CONCLUSIONS

This work proposed the development of an impedance observer for Variable Impedance Actuators. The observer was designed by combining a stiffness observer, designed for non-linear systems, and an Extended Kalman Filter. The clever placement of the torque sensor on the device allowed the decoupling of the two observers, thus avoiding possible instability issues that could have arised from the interaction of the two observation dynamics. The resulting observer was successfully tested on the AwAS variable stiffness actuator, in numerical simulation in a first phase, and in physical experiments subsequently.

VI. ACKNOWLEDGMENTS

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VII. APPENDIX: THE EXTENDED KALMAN FILTER

The EKF is a discrete algorithm to estimated non linear dynamics affected by noise. Given the nonlinear discrete dynamic system

$$\begin{cases} x_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \\ y_k = h(x_k) + v_k \end{cases}, \tag{21}$$

where w_k and v_k are the process and observation noises, both assumed as zero mean multivariate Gaussian noises with covariance matrices Q_k and R_k respectively. Given the Jacobian matrices of the state transition and observation defined as

$$F_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}, u_k} \quad H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k|k-1}}. \tag{22}$$

Starting form the initial conditions $x_{0|0}$ (initial guess) and $P_{0|0}$ (covariance matrix of its likelihood), the EKF iteratively computes the current state prediction and the current estimated covariance matrix of the filter from previous state

$$\begin{aligned} \hat{x}_{k|k-1} &= f(\hat{x}_{k-1|k-1}, u_{k-1}) \\ P_{k|k-1} &= F_{k-1} P_{k-1|k-1} F_{k-1}^T + Q_{k-1}; \end{aligned} \tag{23}$$

then, the algorithm updates the current state estimation from (23) and the current estimation error (first equation of (24))

$$\begin{aligned} \tilde{y} &= r_k - h(\hat{x}_{k|k-1}) \\ K_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k \tilde{y}_k, \end{aligned} \tag{24}$$

where r_k is the current measurement of the non linear system and K_k is the, so called, Kalman filter gain. Finally, the EKF updates the estimation covarianace matrix for the next interaction

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}. \tag{25}$$

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