Decentralized Deployment of Mobile Sensors for Optimal Connected Sensing Coverage

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Abstract. In this paper, we address the *optimal connected sensing coverage* problem, i.e., how mobile sensors with limited sensing capabilities can cooperatively adjust their locations so as to maximize the extension of the covered area while avoiding any internal "holes", areas that are not covered by any sensor. Our solution consists in a *distributed motion algorithm* that is based on an original extension of the Voronoi tessellation.

1 Introduction

Recent technological advances in miniaturization and low–cost production of small embedded devices have greatly enhanced our capability to sense and control the physical world. Wireless Sensor Networks (WSNs) seem to represent one of the main research and application fields that will benefit from these advances, and indeed they are revolutionizing the way data is traditionally gathered from the physical world.

A crucial requirement for an efficient execution of a sensing task is an adequate sensor deployment. In this paper we consider the problem of deploying a set of mobile sensor nodes with limited sensing range in order to achieve an *optimal connected sensing coverage*. Informally, this requires to devise a deployment that maximizes the extension of the covered area while avoiding internal "holes", i.e., internal areas that are not covered by any sensor. We propose a solution to this deployment problem that consists in a *distributed motion algorithm* that makes sensor nodes cooperatively adjust their locations so as to fulfill the requirement of optimal connected sensing coverage.

The deployment of mobile sensor nodes is certainly not new [1, 2, 3]. It is particularly relevant in areas that are remote, hostile or even deadly for human operators, and its employment has been promoted by the availability of mobile sensors such as Robomote [4].

The coverage problem we solve is similar to the one studied in [2,1]. However, they use a mix of fixed and mobile sensors and do not prove the eventual absence of coverage "holes". Our solution extends the one based on Voronoi diagrams proposed by Cortes *et al.* that assumes sensors with unlimited sensing ranges

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although degrading with distance [3]. Intuitively, every sensor is mobile and moves under the effects of two contrasting forces. The one tends to keep a sensor close to its neighbors whereas the other tends to spread mobile sensors as much as possible. The former is the strongest but it is exerted only when the mutual distance between a couple of neighbors exceeds a predefined threshold. The latter is weaker but is always present. In a typical evolution of the system, mobile sensors initially assemble to fill every coverage hole and then try to cover as much area as possible without creating any hole.

The paper is organized as follows. In Section 2.1, we present the theoretical foundation of the proposed distributed motion algorithm for optimal sensor deployment. Then, in Section 2.2, we consider how to achieve a sensor deployment such that neighboring sensors are within a given desired distance and argue about the algorithm convergence toward a steady optimal solution. Then, we discuss how such a distance can be chosen so that a final optimal connected sensor deployment is eventually reached.

2 Distributed Deployment for Sensing Connectivity

2.1 Basic Framework

Consider n sensors that are to be deployed within a configuration space $\mathcal{Q}$. Let us suppose that the current configuration or location q_i of the i–th sensor is measured w.r.t. a common coordinate frame. For the sake of simplicity, we focus on a planar deployment problem, where the configuration space $\mathcal{Q}$ is a closed region $\mathcal{W} \subset \mathbb{R}^2$, and the position of the i–th sensor is $q_i = (x_i, y_i)$, although the discussion remains valid for problems in higher dimensions.

Assume that the desired *sensor distribution* is specified by a non–negative function $\phi : \mathcal{Q} \to \mathbb{R}^+$, whose value at any location q in $\mathcal{Q}$ is proportional to the need of sensing the location itself. Clearly, a null value of $\phi(q)$ means that a sensor is not required at q. Possible sensor distributions may range from a uniform $\phi(q)$, meaning that every location in $\mathcal{W}$ require the same sensing effort, to a spot–wise $\phi(q)$ representing situations where there is one or more discrete points of interest. Consider also a non–negative function $f : \mathcal{Q} \times \mathcal{Q} \to \mathbb{R}^+$ that represents a *sensing degradation*, also referred to as *sensing model*. More precisely, for a fixed position q_i of the i–th sensor, $f(q_i, q)$ is a function of q that describes how the sensor's measurement degrades as q varies from q_i. This is how we will use f in the rest of the paper.

To begin with, suppose that sensors are deployed at initial locations $q_i(0)$, for $i = 1, \ldots, n$. Then, imagine that the configuration space $\mathcal{W}$ is partitioned into n regions based on the current sensors' locations, i.e. $\mathcal{W} = \cup_{i=1}^{n} \mathcal{W}_i$, and $\mathcal{W}_i = \mathcal{W}_i(Q)$, where $Q = \{q_1, \ldots, q_n\}$. It is useful to assume that each sensor becomes responsible for sensing over exactly one region that will be referred to as its *dominance region*. Then, to find an optimal sensor deployment, we need to introduce a global *cost functional* $\mathcal{H}(Q, \mathcal{W})$ that measures how poor is the current sensor deployment w.r.t. the desired sensor distribution $\phi(q)$ for the given sensing model $f(q)$. One possible choice for $\mathcal{H}$ is the following cost functional [3]:

$$\mathcal{H}(Q, \mathcal{W}) = \sum_{i=1}^{n} \int_{\mathcal{W}_i} f(||q - q_i||)\, \phi(q)\, dq\,, \tag{1}$$

where $||\cdot||$ is the Euclidean norm. Let us now consider the following problem:

Problem 1 (Optimal Deployment Without Sensing Connectivity). Given a desired sensor distribution $\phi(q)$, and a sensing degradation $f(q)$, find a *distributed motion strategy* by using which n mobile sensors can iteratively adjust their locations from an initial deployment at $q_i(0)$, $i = 1, \ldots, n$, to a final optimal deployment minimizing $\mathcal{H}$.

This problem has been solved as an instance of *coverage control* by Cortes *et al.* in [3]. Therein, Corte *et al.* assumed that sensors move according to a *motion model* described by a first–order linear dynamics:

$$\dot{q}_i(t) = u_i(t), \quad \text{for } i = 1, \ldots, n\,, \tag{2}$$

where u_i are *input velocities* that can be chosen to move the sensors. This model assumes that mobile sensors can move to any location where they are asked to move. In practice, finding paths or maneuvers that take mobile sensor nodes to desired destinations is an important problem that can easily become difficult when there are obstacles in the field and kinematic constraints. This problem is studied in the area of robotics [5,6] and we do not study it any further. Moreover, Cortes *et al.* assumed an *isotropic degradation model*:

$$f(q, q_i) = ||q - q_i||^2\,, \quad \text{for all } q_i, q \in \mathcal{Q}\,, \tag{3}$$

which depends only on the Euclidean distance between the i–th sensor's position q_i and the sensed location q. They exploit the known fact that, given a current sensors' deployment $\bar{Q}$, the partitioning $W = \{\mathcal{W}_1, \ldots, \mathcal{W}_n\}$ minimizing $\mathcal{H}$ is the *Voronoi tessellation* $\mathcal{V}$ generated by $\bar{Q}$, i.e. $\mathcal{V}(\bar{Q}) = \text{argmin}_{\{\mathcal{W}_1,\ldots,\mathcal{W}_n\}} \mathcal{H}(\bar{Q}, W)$. Intuitively, given n sensors, a Voronoi tessellation is a partition of the environment into n regions, where the i–th region is formed of all locations whose distance from sensor i is less or equal than the distances from other sensors [7]. Furthermore, Cortes *et al.* consider the desired sensor distribution $\phi(q)$ as a *density function* over the domain $\mathcal{Q}$ and recall two important quantities associated with the i–th Voronoi region $\mathcal{V}_i$. In particular, they have used the generalized mass, and the centroid or center of mass of $\mathcal{V}_i$ that are respectively defined as [8]:

$$M_{\mathcal{V}_i} = \textstyle\int_{\mathcal{V}_i} \phi(q)\, dq\,, \; C_{\mathcal{V}_i} = \frac{1}{M_{\mathcal{V}_i}} \int_{\mathcal{V}_i} q\, \phi(q)\, dq\,. \tag{4}$$

Using this physical interpretation of $\phi(q)$, Cortes *et al.* proposed a *distributed gradient–based motion strategy* allowing sensors to improve their deployment and thus reduce $\mathcal{H}$. More precisely, they showed that a motion strategy where each sensor is subject to a force generated by $\phi(q)$ and pushing it toward the centroid $C_{\mathcal{V}_i}$ of its current dominance region $\mathcal{V}_i$ solves Problem 1. Note that all dominance regions $\mathcal{V}_i$s are updated at any instant by every sensor and are thus a function of time, i.e. $\mathcal{V}_i = \mathcal{V}_i(t)$. Hence we have $M_{\mathcal{V}_i} = M_{\mathcal{V}_i}(t)$, and $C_{\mathcal{V}_i} = C_{\mathcal{V}_i}(t)$.

2.2 Distance–Constrained Deployment and Sensing Connectivity

We consider a WSN where each sensor i is able to measure any quantity of interest at any location q laying within sensing range r_s from the sensor position q_i itself. This property can be modeled by the following isotropic sensing degradation with threshold r_s:

$$f_{r_s}(q, q_i) = \begin{cases} ||q - q_i||^2 & ||q - q_i|| \leq r_s \,, \\ \infty & ||q - q_i|| > r_s \,. \end{cases} \tag{5}$$

In this context, we aim at finding distributed motion strategies allowing sensors to achieve *sensing connectivity* within $\mathcal{W}$. After recalling the outward boundary ∂A of a closed set A, and its closure A^* being the set of all locations contained by ∂A, we can readily provide the following definition of connectivity:

Definition 1. *Given a closed region $A \subseteq \mathcal{W}$, a quantity of interest ξ, and a sensor deployment Q, we say that A is* sensing connected *if, and only if, for any location $q \in A^*$, there exists at least one sensor in Q that can measure $\xi(q)$.*

Reaching *sensing connectivity* means that we want sensors to be deployed in such a way that there exists a closed sub–region $\mathcal{W}_c \subseteq \mathcal{W}^*$ that contains no sensing holes and that is as large as possible compatibly with this constraint. Hence, the problem we want to solve becomes the following:

Problem 2 (Optimal Deployment With Sensing Connectivity). In addition to the requirements of Problem 1, find a motion strategy by which sensors can also establish and maintain sensing connectivity.

A distributed motion strategy allowing the sensing connectivity requirement to be fulfilled needs a *form of interaction and coordination* between *neighboring sensors*. Indeed, to solve Problem 2, it is strategically important to choose a suitable definition of neighborhood and a local form of interaction between any two neighboring sensors. Among many possible choices, we will exploit the *neighborhood relation* introduced by the Voronoi tessellation, i.e. we will require that sensors i and j coordinate their motions in order to establish and maintain sensing connectivity if, and only if, they are $\mathcal{V}$–neighbors. Furtheron, to *enforce* the sensing connectivity requirement, we introduce *virtual points of interest*, that modify the originally given density function $\phi(q)$, for any couple of sensors that are too far from each other.

To this aim, first denote with $V = \{v_{ij}\}$ the *adjacency matrix* of a Voronoi graph $G_{\mathcal{V}}$, where the generic element is $v_{ij} = 1$ if i is a $\mathcal{V}$–neighbor of j, or $v_{ij} = 0$ otherwise. Then, consider the *augmented sensing distribution* $\tilde{\phi}$ defined as follows:

$$\tilde{\phi}(q, d) = \phi(q) + \alpha \sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij} \, \frac{c_d(||q_i - q_j||)}{2} \, \delta\left(q - \frac{q_i + q_j}{2}\right) , \tag{6}$$

where $\alpha \in \mathbb{R}$ is a positive *weight* that is chosen such that $\alpha \gg \max_{q \in \mathcal{W}} |\phi(q)|$, $c_d(s)$ is a local *penalty* function, and $\delta(q)$ is Dirac's delta function. Function $c_d(s)$

should be chosen so as to penalize pair of sensors at larger relative distances then a suitable *neighborhood threshold* d. One possible choice is the following:

$$c_d(s) = \begin{cases} 0 & \text{if } 0 \le s \le d\,, \\ e^{s-d} - 1 & \text{if } s \ge d\,. \end{cases} \tag{7}$$

Virtual points of interest act as *contracting terms*, such as nonlinear springs, activating whenever the distance between any two $\mathcal{V}$–neighbors exceeds a certain threshold. Their action is weighted by α which is chosen to be much larger than the maximum value of $\phi(q)$. This means that the sensing connectivity requirement has higher "priority" than the WSN's deployment task. We can draw an *analogy of our strategy with a particle system* where particles are subject to an external force generated by potential field ϕ, but are also aggregated by "stronger" internal forces as for the Van der Waals phenomenon.

On the way to solve Problem 2, we will proceed by finding a distributed motion strategy by which the distance between any two $\mathcal{V}$–neighbors is constraint to be less or equal to d. On the same line of [3], we will try to minimize the following global cost functional $\tilde{\mathcal{H}}$ that measures how poor is a sensor deployment Q w.r.t. the augmented density $\tilde{\phi}$ for the sensing model f_{r_s}:

$$\tilde{\mathcal{H}}(Q, \mathcal{W}, d) = \sum_{i=1}^{n} \int_{\mathcal{W}_i} f_{r_s}(||q - q_i||)\, \tilde{\phi}(q, d)\, dq\,. \tag{8}$$

Our strategy will be to minimize $\tilde{\mathcal{H}}(Q, \mathcal{W}, d)$ by following a gradient–based motion. It is worth noting that the discontinuity of f_{r_s}, that occurs whenever the distance between q_i and q exceeds r_s, may represent a problem for the algorithm. However, it can be shown that stationary configurations of the WSN for f_{r_s} are also stationary for f, and thus we will use the continuous function f in place of f_{r_s}. Having said this, we are ready to state the following first result [9]:

Theorem 1 (Optimal Distance Constraint Deployment). *Consider a WSN where sensors move according to the linear motion model of Eq. 2. Then, given a desired neighborhood distance d, the* distributed motion strategy *described by:*

$$u_i(t) = -k\left(q_i(t) - \tilde{C}_{\mathcal{V}_i(t)}\right)\,, \quad \textit{for } i = 1, \ldots, n\,, \tag{9}$$

where k is a positive real constant, and

$$\begin{aligned} \tilde{C}_{\mathcal{V}_i}(t) &= \tfrac{M_{\mathcal{V}_i(t)}}{\tilde{M}_{\mathcal{V}_i(t)}} \left(C_{\mathcal{V}_i(t)} + \alpha \textstyle\sum_{i,j=1}^{n} v_{ij}(t)\, \frac{c_d(||q_i(t)-q_j(t)||)}{2}\, \frac{q_i(t)+q_j(t)}{2}\right)\,, \\ \tilde{M}_{\mathcal{V}_i}(t) &= M_{\mathcal{V}_i(t)} \left(1 + \alpha \textstyle\sum_{i,j=1}^{n} v_{ij}(t)\, \frac{c_d(||q_i(t)-q_j(t)||)}{2}\right)\,, \end{aligned} \tag{10}$$

where $M_{\mathcal{V}_i(t)}$ and $C_{\mathcal{V}_i(t)}$ are computed as in Eq. 4, makes it possible to reach an optimal sensor deployment where any two $\mathcal{V}$–neighbors are within distance d from each other, and the extension of the covered region is maximized.

The proof of Theorem 1 is omitted for the sake of space and can be found in [9] along with an early implementation and a performance evaluation of a protocol realizing the specified motion strategy.

Finally, we can show that a suitable choice of d makes it possible to achieve the sensing connectivity requirement. Let us denote with I the index set of all *internal* Voronoi regions, being those regions $\mathcal{V}_i$ having all of its faces adjacent to other regions $\mathcal{V}_j$, $j \neq i$, and not to the boundaries of the considered region $\mathcal{W}$. Then, we can prove the following result [9], which solves Problem 2:

Theorem 2 (Optimal Connected Sensing Deployment). *A set of n sensors moving according to the distributed motion strategy of Equation 9, where the neighborhood threshold is chosen as $d = \sqrt{3}\, r_s$, eventually reach a final deployment such that the union of all internal Voronoi regions, $\mathcal{W}_c = \cup_{i \in I} \mathcal{V}_i$, forms an optimal sensing–connected region.*

3 Conclusion

In this paper we studied the problem of reaching optimal connected sensing coverage and proposed a fully distributed Voronoi–based motion strategy for a mobile WSN. Future work will address implementation of the motion strategy and evaluate the robustness of the obtained solution w.r.t. possible message loss.

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