# Allometry

Smart and biomimetic materials

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Ermes Botte

*ermes.botte@phd.unipi.it*

Centro E. Piaggio
bioengineering and robotics research center

# *In Vitro* Models (IVM) group

**Arti AHLUWALIA, full Professor**

Chiara MAGLIARO, Post-Doc researcher

Ludovica CACOPARDO, Post-Doc researcher

Joana COSTA, Post-Doc researcher

Daniele POLI, Post-Doc researcher

Roberta NOSSA, PhD student

Ermes BOTTE, PhD student

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IVMgroupUNIPI

ivmgroup-centro-piaggio

## BIOREACTORS

DESIGN AND REALIZATION

COMPUTATIONAL MODELS

SENSING AND ACTUATION

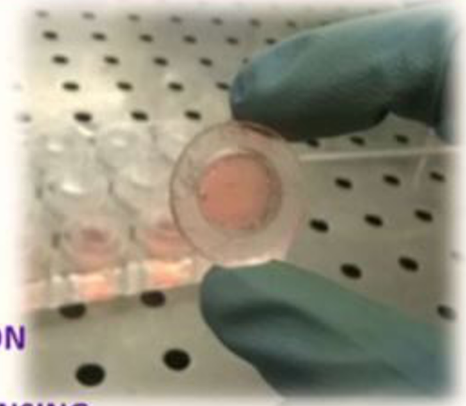
## BIOMECHANICS & BIOMATERIALS

HYDROGELS & BIOPRINTING

TISSUE-DERIVED SCAFFOLDS

MECHANICAL CHARACTERIZATION

CELL MECHANOSENSING



## CELL IMAGING

TISSUE DELIPIDATION

CELL MORPHOMETRICS

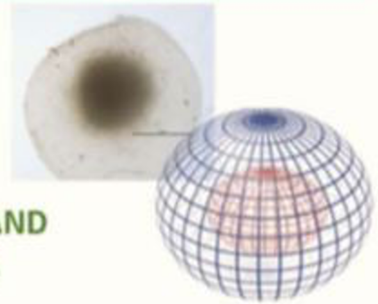
IMAGE PROCESSING

## LIVER AND BRAIN ORGANOIDS

ALLOMETRIC SCALING

NANOTOXICOLOGY

NUTRIENT TRANSPORT AND CONSUMPTION MODELS



**Allometry**: scaling property emerging in all living organisms about characteristic physiological parameters (*e.g.* metabolic rates), which are related to body size (*i.e.* mass) through power laws.

$$Y = aM^b$$

$$\log Y = \log a + b \log M$$

- *a* normalization constant (depending on *Y* and on taxonomic class)
- *b* scaling exponent (depending on *Y*)

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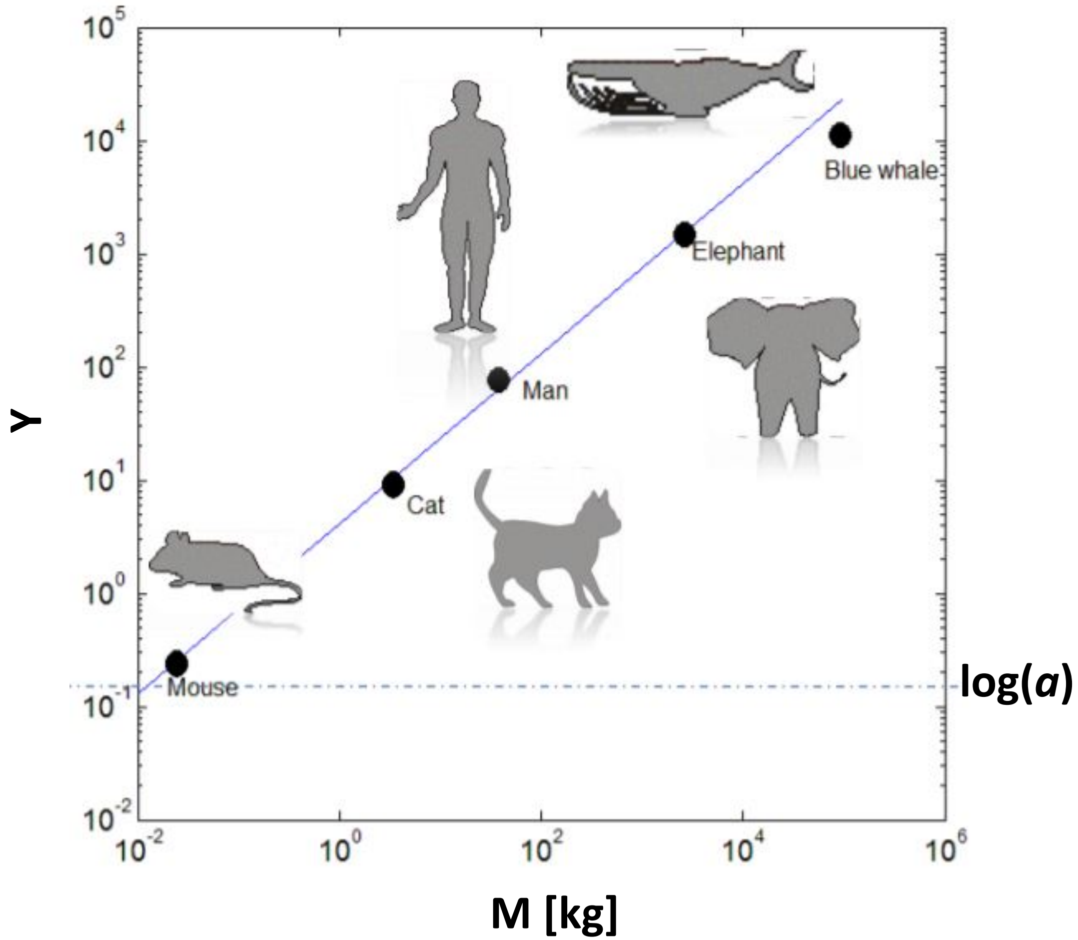


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| PARAMETER | EXPONENT VALUE | MEANING |
| --- | --- | --- |
| Cells size [m]<br>Blood velocity [m/s]<br>Pressure gradients [Pa] | b = 0 | Parameter and body mass are indipendent |
| Volumes (bone, blood…) [m^3] | b = 1 | Parameter and body mass are directly proportional (isometric scaling) |
| Metabolic rates [J/s]<br>Flow rates (haematic, respiratory…) [m^3/s] | b = 3/4 | Parameter increases slower than body mass |
| Radii of aorta and trachea [m] | b = 3/8 | Parameter increases slower than body mass |
| Frequencies (cardiac, respiratory…) [Hz] | b = - 1/4 | Parameter decreases when body mass increases |
| Bone mass [kg] | b = 4/3 | Parameter increases faster than body mass |

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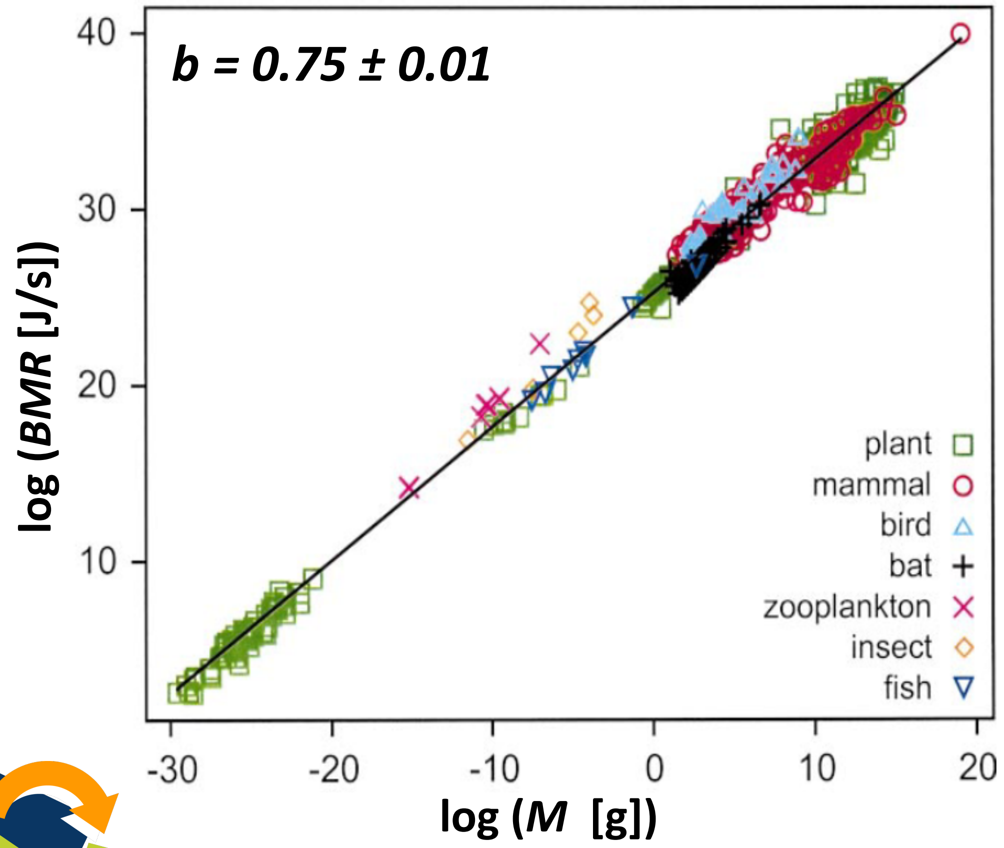
# Kleiber law (KL)

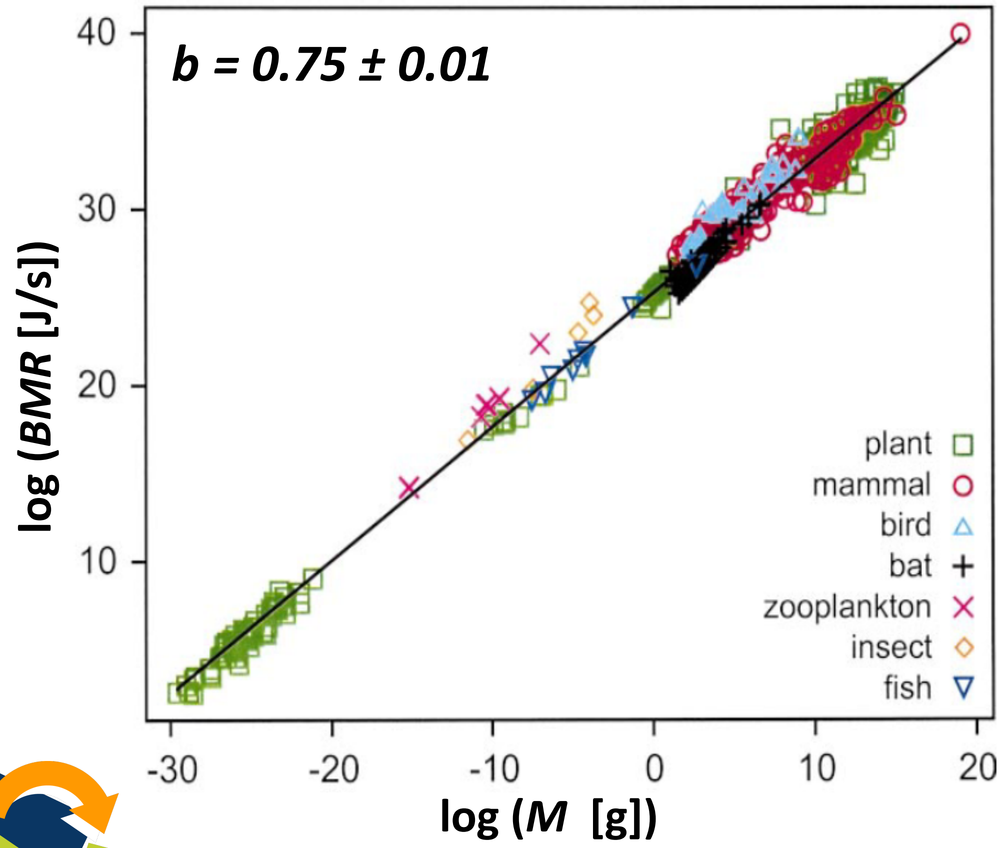
$$BMR = aM^{3/4}$$

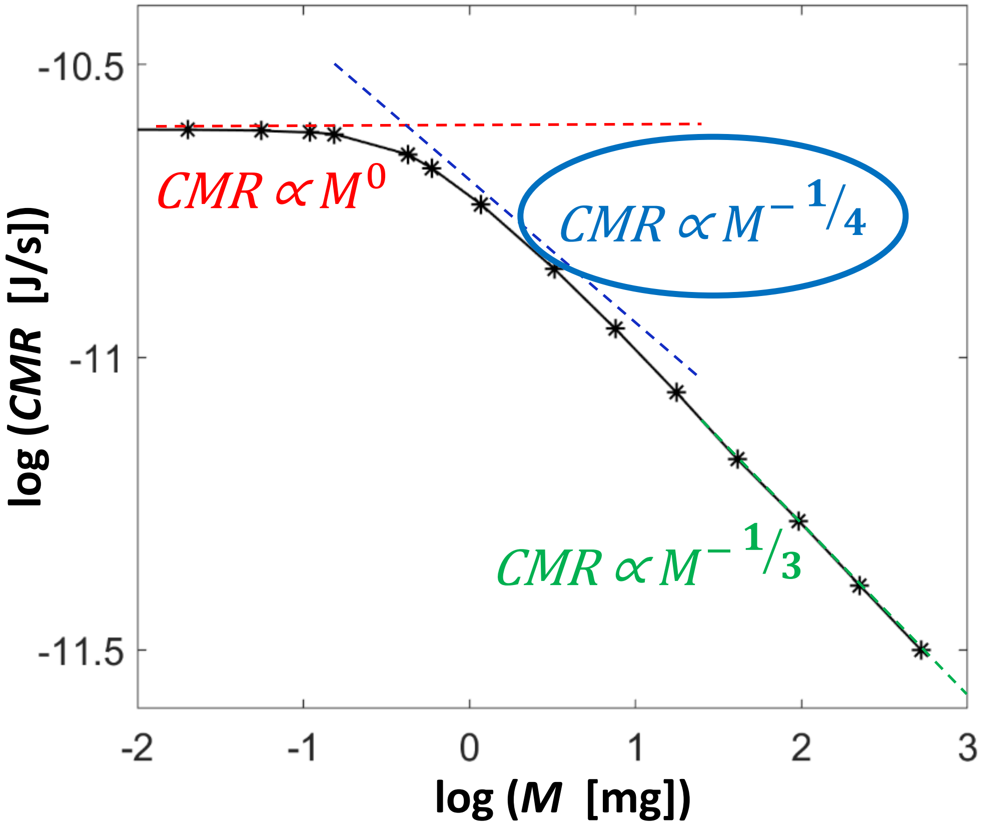


# Kleiber law (KL)

$$BMR = aM^{3/4}$$

$$CMR = a'M^{-1/4}$$

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# Origin of quarter-power scaling

- Geometric scaling

$$\begin{cases} A \propto l^2 \\ V \propto l^3 \\ V \propto M \end{cases} \Rightarrow \quad \boldsymbol{l \propto M^{1/3}} \qquad \boldsymbol{A \propto M^{2/3}}$$

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Why allometric exponents are not multiples of 1/3?

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- **Biological scaling**
  - Stoichiometric constraints in biochemical processes
  - Integrated optimization of interdependent sub-systems
  - Self-similar structure of nutrients supply networks

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- Biological scaling
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Since **metabolic rates** (per cell, *i.e.* CMR) **underlie all physiological processes** and scale with $M^{-1/4}$, allometry is described by quarter-power scaling

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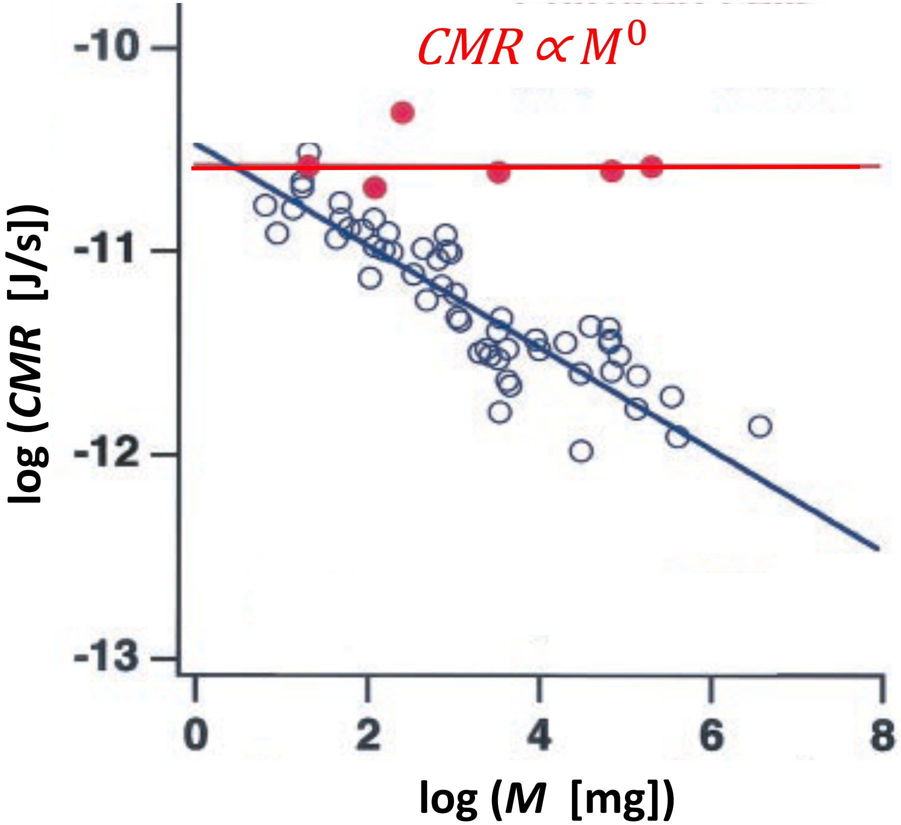
# Why allometry in bioengineering?





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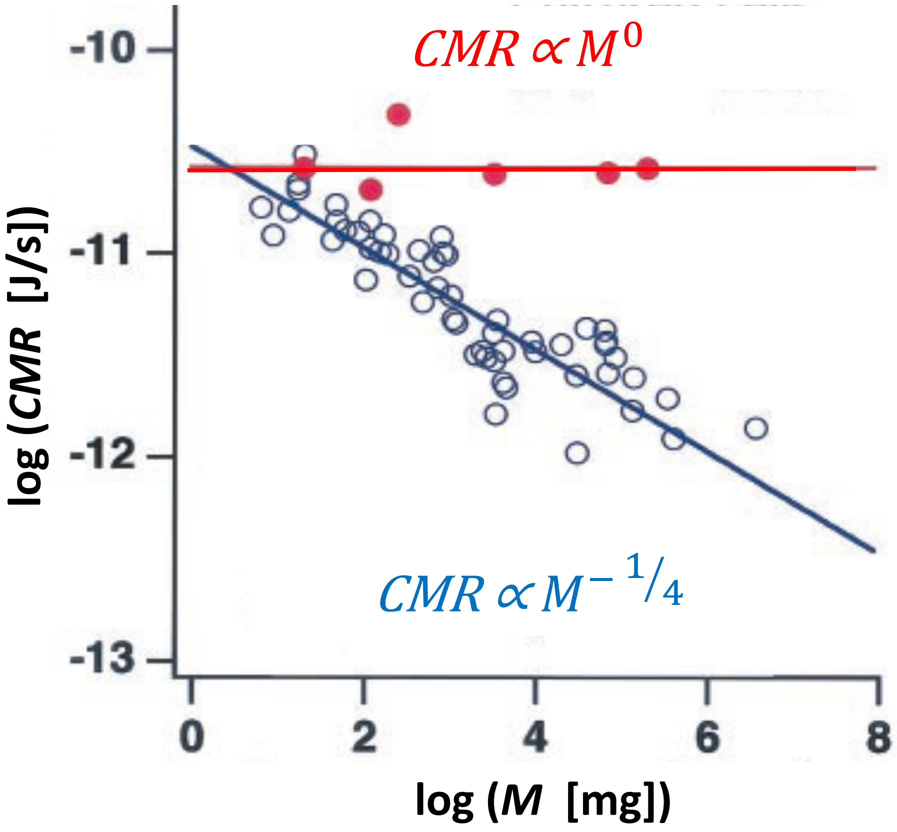
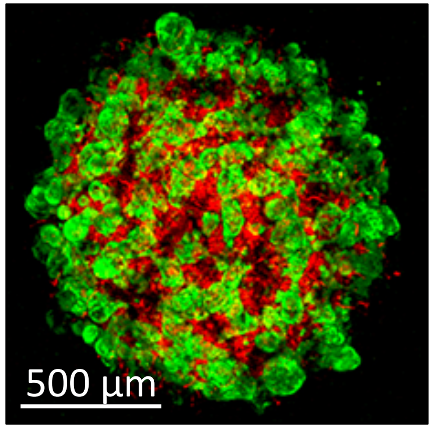



***In vitro* monolayers**

***In vivo***

**3D *in vitro* models**





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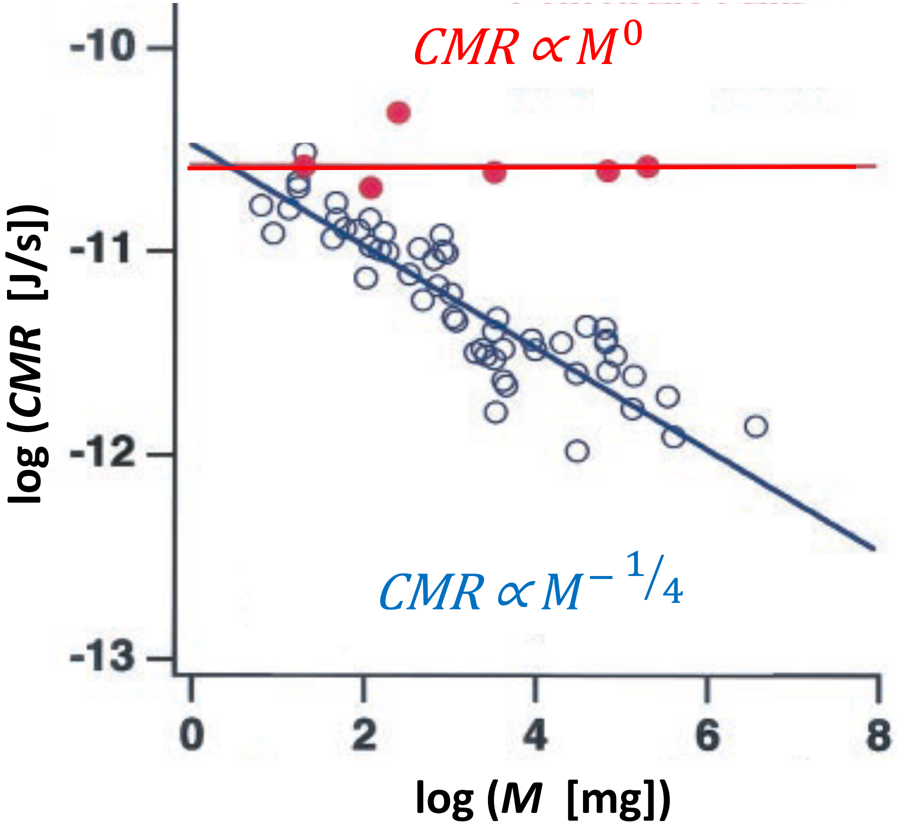
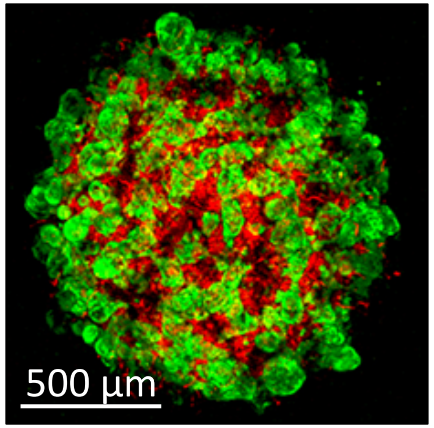



*In vitro* monolayers

*In vivo*
3D *in vitro* models




Since **KL** is a **universal law for biological systems,** we need to account for it to design predictive and **physiologically relevant *in vitro* models**!

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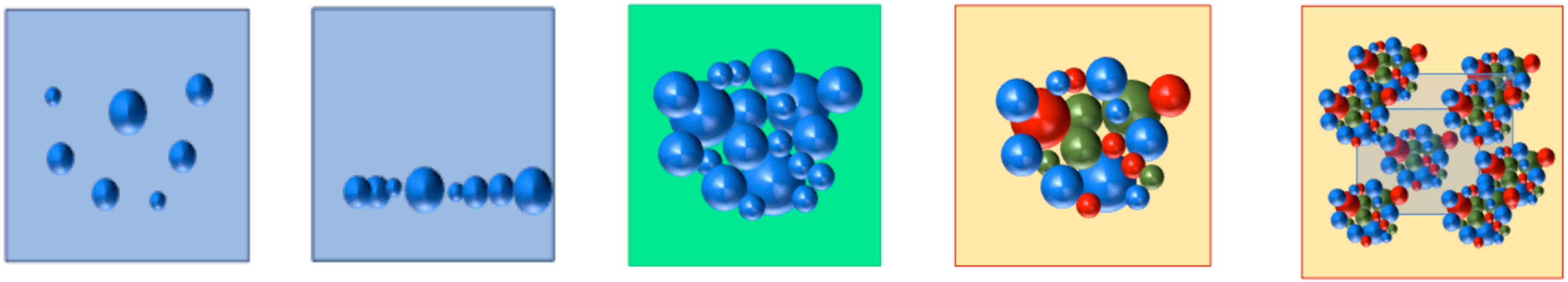
# Why allometry in bioengineering?

Single cells | 2D monolayers | Spheroids | Organoids | 3D architectures of spheroids/organoids



**INCREASING COMPLEXITY**

Which size level does allometric scaling start from?

Which is the size range allowing allometric scaling to emerge?

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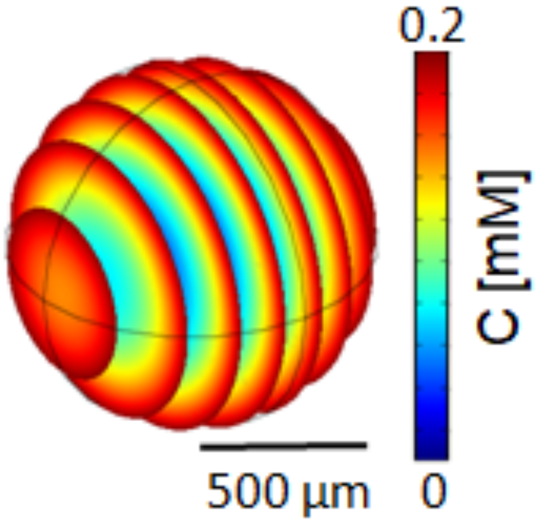
# Case of study: liver and brain *in vitro* models

***In silico*** **modelling: diffusion and consumption**

$$\frac{\partial c_{O_2}}{\partial t} = \nabla^2\left(D_{O_2} c_{O_2}\right) + R = \nabla^2\left(D_{O_2} c_{O_2}\right) - \frac{V_{max} c_{O_2}}{k_M + c_{O_2}}$$

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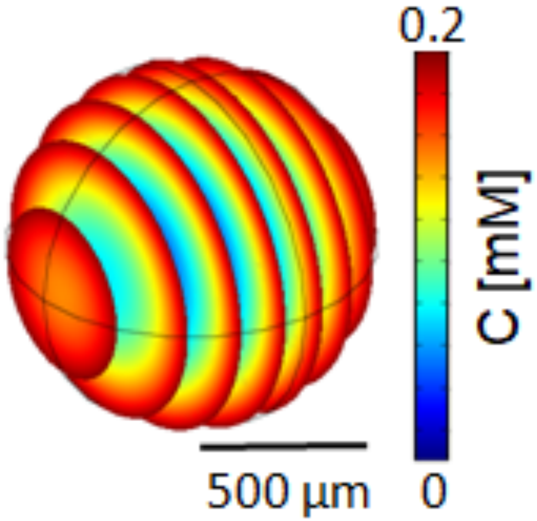


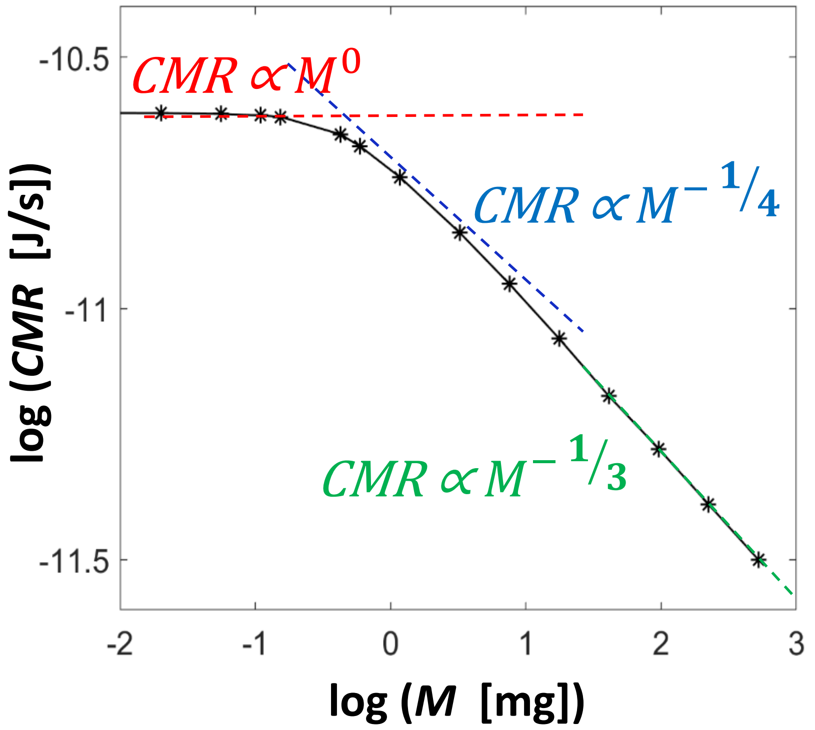


$$B_{O_2} = -\Delta H \oiint \underline{J}_{O_2} \cdot \partial \underline{A} = -\Delta H \oiint \underline{\nabla}\left(D_{O_2} c_{O_2}\right) \cdot \partial \underline{A}$$

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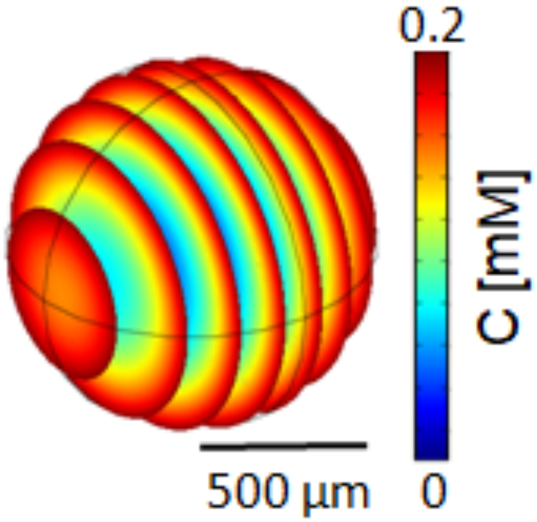


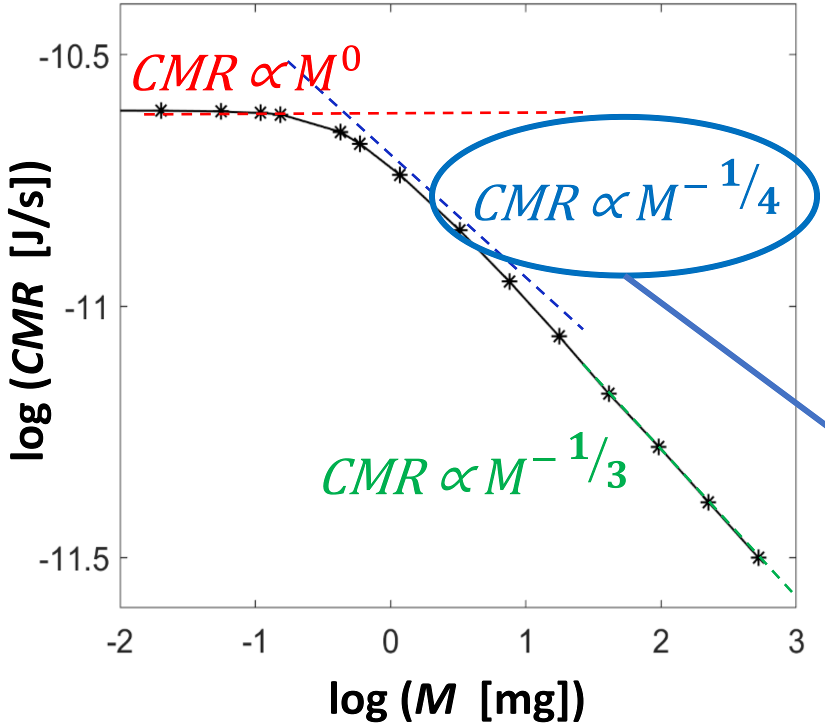


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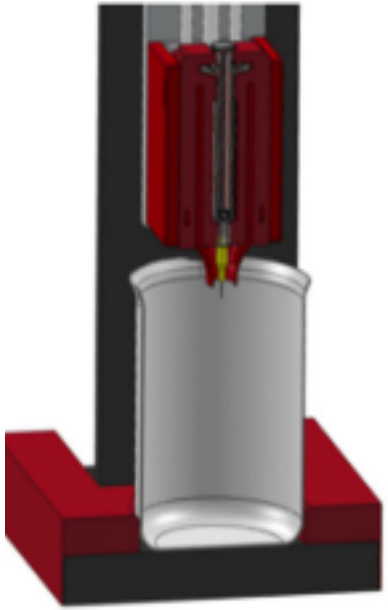
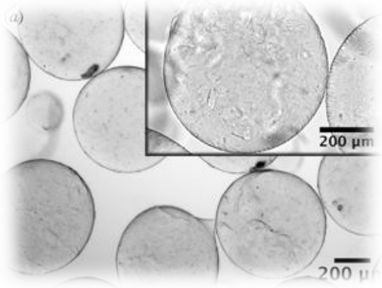
Fabrication, *in vitro* **$O_2$ measurements** and **validation**



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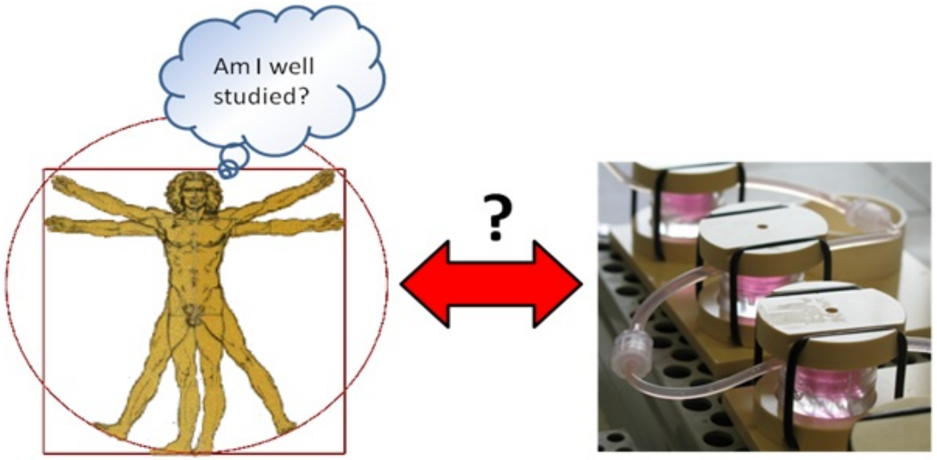
# *Thesis are available on these topics!*




If you are interested in:

Prof. Arti Ahluwalia – *arti.ahluwalia@unipi.it*

Chiara Magliaro – *chiara.magliaro@googlemail.com*

Ermes Botte – *ermes.botte@phd.unipi.it*

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