Visual-servoed Parking with Limited View Angle

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Abstract. In this paper the problem of stabilizing a wheeled vehicle of unicycle type to a set point, using only visual feedback, is considered. The practically most relevant problem of keeping the tracked features in sight of the camera while maneuvering to park the vehicle is taken into account. This constraint, often neglected in the literature, combines with the nonholonomic nature of the vehicle kinematics in a challenging controller design problem. We provide an effective solution to such problem by using a combination of previous results on non-smooth control synthesis, and recently developed hybrid control techniques. Simulations and experimental results on a laboratory vehicle are reported, showing the practicality of the proposed approach.

1 Introduction

In the literature, problems concerning mobile robots stabilization have received wide attention. Set-point stabilization (a problem that for a few years challenged the research community, due to the famous theorem of Brockett on smooth stabilization [2]), has been solved (assuming full state information) among others by [3,7,4,15] using time-varying control laws, and by e.g. [5,6] by non-smooth feedback control laws.

In practical applications of automated vehicles, however, one is confronted with the problem of knowing the current position and orientation of the vehicle only through indirect measurements by available sensors. Although much work has been done on techniques for vehicle localization based on combinations of sensory information (odometry, laser range finders, cameras, etc.), very little is known about the real time connection of a localization algorithm and a feedback control law. In this paper, we consider a vehicle only equipped with a fixed monocular camera, and control laws that close the feedback loop directly at the sensor level, i.e. using information available from 2D images, with very limited information on the environment. Previous works on visual servoing of nonholonomic vehicles include those of [12] and [8]. In the latter papers a feedback control law stabilizing the vehicle posture by using visual information only was solved. Furthermore, [8] considers the practically most relevant problem of keeping the features to be tracked within sight of a limited aperture camera while the vehicle maneuvers to park, and proposes a heuristic correction to the control law that solves the problem in many cases.

The limited-field-of-view constraint combines with the nonholonomic nature of the vehicle kinematics in a challenging controller design problem. In the visual servoing literature the problem of the limited-field-of-view has been considered in [13,14], who used movable cameras to allow target tracking. In this paper, we provide an effective solution to the problem with a camera fixed on board the

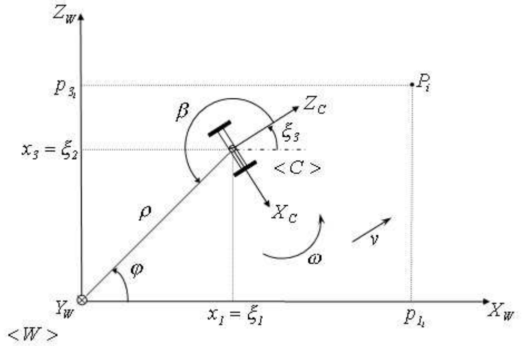

Fig. 1. Fixed frame $< W >$, camera frame $< C >$, and relative coordinates (ξ_1, ξ_2, ξ_3) and (ρ, ϕ, β)

vehicle, by using a combination of previous results on non-smooth control synthesis and recently developed hybrid control techniques. Simulations and experimental results on a laboratory vehicle are reported, showing the practicality of the proposed approach.

2 Feedback Control Law

Consider first a moving frame $< C >$, fixed on the mobile robot with the origin in the camera pinhole, and the Z_c axis directed along the camera optical axis (see fig. 1). The vehicle starts in an unknown position, grabbing an image of a portion of the scene in view. From the grabbed image, $n \geq 2$ characteristic points (features) are selected. Under the assumption that the motion is planar, the coordinates ${}^cY_i = h_i$ of each feature are constant and represent the height of the feature on the plane of motion. By inverting the perspective projection, it is possible to reconstruct the position of the features in the camera frame $< C >$ from the corresponding image points and to estimate the image Jacobian (see [8]).

Consider a fixed frame $< W >$ whose origin is coincident with the origin of $< C >$ when the robot is in the desired final configuration, and with $X_w = Z_c$ and $Y_w = Y_c$. Let ${}^W\xi = {}^W[\xi_1, \xi_2, \xi_3]^T \in \mathbb{R}^2 \times S$ denote the robot posture. More precisely, (ξ_1,ξ_2) are the cartesian coordinates of the middle point of the unicycle axle, and ξ_3 is the orientation of the unicycle between the Z_c axis and the X_w axis, as represented in figure 1. Let ${}^WP_i = {}^W[p_1, p_2, p_3]_i^T$ be the i–th feature coordinates. All features are motionless in $< W >$. Knowing the current position of the features in the camera frame $< C >$, assuming a number of features $n \geq 2$ and that their height ${}^Cp_2^i = h_i$ is constant during the planar motion, the actual unknown position and orientation ${}^{W}\xi$ of the unicycle can be evaluated by solving (in a least-squares sense) for (ξ_1, ξ_2, ξ_3) the following equation:

$$ {}^{C}\begin{bmatrix} p_1^1 \\ p_3^1 \\ \vdots \\ p_1^n \\ p_3^n \end{bmatrix} = \begin{bmatrix} {}^{W}p_3^1 & {}^{W}p_1^1 & 1 & 0 \\ -{}^{W}p_1^1 & {}^{W}p_3^1 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ {}^{W}p_3^n & {}^{W}p_1^n & 1 & 0 \\ -{}^{W}p_1^n & {}^{W}p_3^n & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\xi_3 \\ \sin\xi_3 \\ \xi_2\cos\xi_3 - \xi_1\sin\xi_3 \\ -\xi_1\cos\xi_3 - \xi_2\sin\xi_3 \end{bmatrix}. \tag{1} $$

This information can be obtained using only image data (*Image Position Based Controller*).

The main practical constraint in the design of a control law is related to the necessity of tracking features continuously. Indeed, while it is in principle possible to recover from a feature loss by replanning or by resorting to other sensorial information (such as e.g. odometry), this is usually to be avoided in the interest of simplicity and robustness.

We first consider the design of a control law for a simplified problem, that is the stabilization of the unicycle in the desired position and orientation with the only constraint that a single feature, placed at the origin of the desired frame $< W >$, is kept in view. To solve this particular problem, it is expedient to refer the control law design to suitable coordinates, allowing to incorporate more easily the information about the position of the selected feature with respect to the camera.

Consider hence a change of coordinates $\Phi : \mathbb{R}^2 \times S \to \mathbb{R}^+ \times S^2$ with

$$ \begin{bmatrix} \rho \\ \phi \\ \beta \end{bmatrix} = \Phi(\xi_1, \xi_2, \xi_3) = \begin{bmatrix} \sqrt{{\xi_1}^2 + {\xi_2}^2} \\ \arctan(\frac{\xi_2}{\xi_1}) \\ \pi + \arctan(\frac{\xi_2}{\xi_1}) - \xi_3 \end{bmatrix} \tag{2} $$

(see fig. 1). Observe that this change of coordinates is a diffeomorphism everywhere except at the origin of the plane, as discussed in more detail in [5].

The dynamics of the vehicle in the new coordinates are easily obtained as

$$ \begin{bmatrix} \dot\rho \\ \dot\phi \\ \dot\beta \end{bmatrix} = \begin{bmatrix} -\rho\,\cos\beta \\ \sin\beta \\ \sin\beta \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \omega, \tag{3} $$

where we let $u = \frac{v}{\rho}$. Consider the candidate Lyapunov function (proposed in [5]) $V = \frac{1}{2}(\rho^2 + \phi^2 + \lambda\beta^2)$, with $\lambda > 0$ a free parameter to be used in the following controller design. One has

$$ \dot V = -\rho^2\cos\beta\; u + \phi\sin\beta\; u + \lambda\beta\sin\beta\; u - \lambda\beta\omega, \tag{4} $$

and, by setting

$$ u = \cos\beta, \text{ and } \omega = \frac{\phi\sin\beta\cos\beta + \lambda\beta\sin\beta\cos\beta}{\lambda\beta} + \beta, \tag{5} $$

one gets $\dot{V} = -\rho^2 \cos^2\beta - \lambda\beta^2 \leq 0$. By using Lasalle's invariant set theorem, the asymptotic stability of the origin is proved. The controlled dynamics are then

$$\begin{bmatrix} \dot{\rho} \\ \dot{\phi} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\rho \cos^2\beta \\ \sin\beta\cos\beta \\ -\frac{\phi \ \sin 2\beta}{2\lambda\beta} - \beta \end{bmatrix}. \tag{6}$$

Observe that this stabilization strategy coincides so far with that proposed by ([5]).

Let the limited field-of-view be described by a symmetric cone centered in the optical axis Z_c with semi-aperture Δ. The feature-tracking constraint (for a single feature in the origin) is thus simply $|\beta| < \Delta$. Consider now the positive definite function $V(\phi,\beta) = \frac{\phi^2}{2} + \frac{\lambda\beta^2}{2}$, and an ellipse in the plane (ϕ,β) defined by $V \leq \frac{\lambda\Delta^2}{2}$. Along the systems trajectories, one has $\dot{V} = -\lambda\beta^2$ which is negative semi-definite. Therefore, if the initial condition (ϕ_0,β_0) is within the ellipse, the evolution of (ϕ,β) remains indefinitely inside the ellipse. For any initial condition (ϕ_0,β_0) such that the feature is visible (i.e., $\beta_0 < \Delta$), it is now possible to choose a value for λ such that the evolution of β is always strictly bounded in the sector $(-\Delta,\Delta)$. Indeed, this can be obtained by putting $\lambda = \sqrt{\frac{{\phi_0}^2}{\Delta^2 - {\beta_0}^2}}$. The actual choice of λ will be made depending on the extent of the expected range of possible initial conditions.

We now consider the case that the feature to be tracked is in a generic position R on the ${X_w}^+$ axis. The task is again to achieve docking at the origin with horizontal heading, by always maintaining the tracked feature in view. For simplicity's sake, we replace the state-space coordinate ϕ with $\alpha = \phi - \pi$, with $\dot{\alpha} = \dot{\phi}$. The constraint on the angle under which the camera views the tracked feature is now written as

$$\gamma(\rho,\alpha,\beta) = \alpha - \beta - \arctan\frac{\rho\sin\alpha}{R + \rho\cos\alpha} \leq \Delta. \tag{7}$$

The application of the control law above described is not sufficient to solve this more complex problem, as the limitation of β through the choice of λ is no longer sufficient to ensure the field-of-view constraint (7). Indeed, with the control law (5), it is only possible to guarantee our goal (i.e., that the system is stabilized to the desired configuration while the field-of-view constraint is not violated), for a particular set $\Sigma \subset \mathbb{R}^+ \times S^2$ of initial conditions.

A description of Σ can be obtained by upper-bounding the absolute value of γ along the path defined by the stabilized dynamics (6), and reasoning along the lines above in the (α,β) plane. As a result, one gets the sufficient region depicted in fig. 2 which is comprised of initial configurations such that the whole X_w axis is within the camera's field-of-view, but is such that the desired configuration itself is on the boundary of Σ. To overcome this limitation and obtain a stabilizing law that can be applied to any initial configuration, provided only that the field-of-view constraint is initially satisfied, we therefore use a more complex hybrid controller described in the next section.

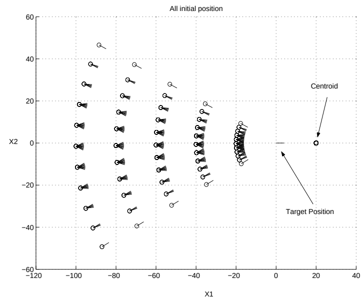

Fig. 2. The set of initial conditions from which the vehicle can be stabilized to the origin by a smooth controller without loosing track of a feature placed on the X_w axis, does not contain the origin in its interior.

3 Hybrid Controller

The basic idea to be applied in this section is rather simple, and is based on the fact that the Lyapunov-based control described in the previous section is not uniquely defined. Rather, a whole family of controllers can be defined by simply redefining the control Lyapunov function candidate (see e.g. [9]). It can be expected that for such different candidates, the resulting stabilizing control laws and ensuing trajectories are different, and that switching among these control laws should be enabled when the field-of-view constraint is about to be violated. To precisely describe this approach, and to prove that it indeed produces a solution to our problem, we formalize the system using tools from the recently developed theory of *hybrid systems*. In their most simple description, hybrid systems are dynamical systems comprised of a finite state automaton, whose states correspond to a continuous dynamic evolution, and whose transitions can be enabled by particular conditions (*guards* or *jumps*) reached at by the continuous dynamics themselves ([1]).

Consider the following five control Lyapunov functions

$$\begin{aligned} V_1(\rho,\alpha,\beta) &= \tfrac{\rho^2}{2} + \tfrac{\alpha^2}{2} + \tfrac{\beta^2}{2}, \\ V_2(\rho,\alpha,\beta) &= \tfrac{\rho^2}{2} + \tfrac{(\alpha-\pi)^2}{2} + \tfrac{(\beta-\pi)^2}{2}, \\ V_3(\rho,\alpha,\beta) &= \tfrac{\rho^2}{2} + \tfrac{(\alpha+\pi)^2}{2} + \tfrac{(\beta-\pi)^2}{2}, \\ V_4(\rho,\alpha,\beta) &= \tfrac{\rho^2}{2} + \tfrac{(\alpha-\pi)^2}{2} + \tfrac{(\beta+\pi)^2}{2}, \\ V_5(\rho,\alpha,\beta) &= \tfrac{\rho^2}{2} + \tfrac{(\alpha+\pi)^2}{2} + \tfrac{(\beta+\pi)^2}{2}. \end{aligned} \tag{8}$$

These definitions can be regarded as generated by regarding the target configuration $\Phi_{goal} \in \mathbb{R}^+ \times S^2$ in five different ways, i.e. respectively $\Phi_{goal} = (0,0,0)$, $\Phi_{goal} = (0,\pi,\pi)$, $\Phi_{goal} = (0,-\pi,\pi)$, $\Phi_{goal} = (0,\pi,-\pi)$ and $\Phi_{goal} = (0,-\pi,-\pi)$. While the five definitions obviously coincide, they engender different ways of reaching at the equilibrium ([9]). For compactness of notation, consider parameter vectors $\tilde{\beta} = [\beta, \beta-\pi, \beta-\pi, \beta+\pi, \beta+\pi]$, and $\tilde{\alpha} = [\alpha, \alpha-\pi, \alpha+\pi, \alpha-\pi, \alpha+\pi]$, so that (8) can be rewritten as $V_i(\rho,\alpha,\beta) = \frac{\rho^2}{2} + \frac{\tilde{\alpha}_i^2}{2} + \frac{\tilde{\beta}_i^2}{2}$. All the above Lyapunov functions have the same first term $\frac{\rho^2}{2}$ whose time derivative can be made non-positive by setting $u = \cos\beta$. The condition that also the second term of each Lyapunov function is negative semidefinite implies that the ω control is chosen as

$$\omega_i = \lambda\tilde{\beta}_i + \frac{\sin\beta\cos\beta}{\tilde{\beta}_i}(\tilde{\alpha}_i + \tilde{\beta}_i), \tag{9}$$

with $\lambda > 0$ a constant parameter to be chosen. These five different control laws (parameterized by λ) define in turn five different controlled dynamics (analogous to (6)) that are globally asymptotically stable in the state manifold $\mathbb{R}^+ \times S^2$, although none of these alone can guarantee that the field-of-view constraint is satisfied throughout the parking maneuver.

A hybrid system with five discrete states is defined by the five dynamics laws above along with a switching law described by a jump event, a commutation law, and an update law for λ (which could be formally regarded as an additional continuous state with trivial dynamics $\dot{\lambda} = 0$ and a reinitialization law at each control commutation).

The jump condition is triggered when, during the stabilization with one of the five control laws, the feature approaches the border of the field of view by a threshold $\Delta_j < \Delta$, i.e. when $|\gamma| \geq \Delta_j$, where γ is defined in (7).

When the jump condition is triggered, a choice is made among other available control functions (hybrid states) according to which guarantees that the feature is brought back towards the optical axis of the camera. In other terms, the control function is commuted to one such that $\gamma\dot{\gamma} < 0$. If multiple such choices exist, the one that maximizes some merit function is picked (in particular, we adopted the criterion that the control law ω_i is the one minimizing $\dot{V}_i$, although this choice is not of major consequence to the convergence of the method).

We first prove that the hybrid controller is deadlock free. Indeed, if along the evolution of the system (with any of the five controllers) γ does not reach the threshold value, then asymptotic convergence to the target configuration is granted by construction. On the other hand, it can be easily shown that in any configuration, there are always at least two possible different control laws among the five such that (by suitably setting λ), one has $\gamma\dot{\gamma} < 0$.

Indeed, by differentiating (7), one has $\dot{\gamma} = \eta - \omega$, with

$$\eta = \frac{\rho\cos\beta}{(R + \rho\cos\alpha)^2 + \rho^2\sin^2\alpha}[R\sin(\beta - \alpha) + \rho\sin\beta].$$

When $\gamma = \Delta_j$, the condition $\dot{\gamma} < 0$ is enforced if a commutation is made to a control function for which it holds

$$\lambda\tilde{\beta}_i > \eta - \frac{\sin\beta\cos\beta}{\tilde{\beta}_i}(\tilde{\alpha}_i + \tilde{\beta}_i). \tag{10}$$

Considering that, for any value of β, among the $\tilde{\beta}_i, i = 1, \ldots 5$ there are at least two positive values, it will be sufficient to choose one of them and a new λ such that

$$\lambda > \frac{\eta}{\tilde{\beta}_i} - \frac{\sin\beta\cos\beta}{\tilde{\beta}_i^2}(\tilde{\alpha}_i + \tilde{\beta}_i), \tag{11}$$

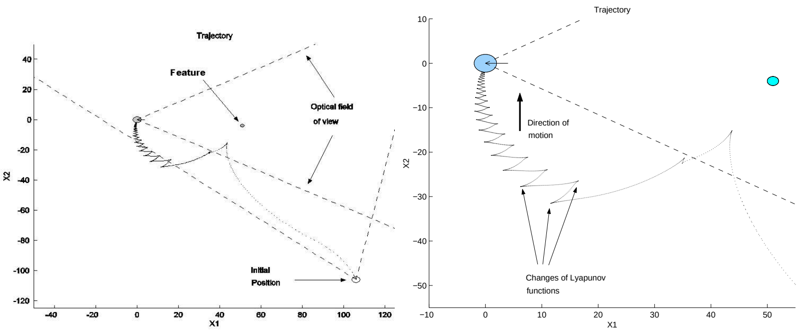

Fig. 3. Left: Trajectory in the fixed frame $< W >$ of the unicycle. The restricted optical field of view of the camera and the starting position of the vehicle are represented. Right: zoom on the desired position

to ensure the condition on $\dot{\gamma}$. The case $\gamma = -\Delta_j$ is similar.

By the above argument, we have that the proposed hybrid control law guarantees satisfaction of the constraint provided only that it is satisfied at the initial configuration. It is now necessary to prove the convergence of the entire system to the desired configuration. Along the hybrid evolution, the distance of the vehicle from the origin, $\rho(t)$, is continuous with continuous first derivative, and non increasing (indeed, $\dot{\rho} = -\rho \cos^2 \beta$ in all states). The set in which $\dot{\rho} = 0$ is given by

$$C_\rho = C_\rho^1 \cup C_\rho^2 = \{(\rho, \tilde{\alpha}_i, \tilde{\beta}_i) : \rho = 0\} \cup \{(\rho, \tilde{\alpha}_i, \tilde{\beta}_i) : \tilde{\beta}_i = \frac{\pi}{2} + k\pi\}.$$

However, it is easy to check that under the controlled dynamics and commutation laws above described, all configurations in C_ρ^2 are not invariant. Hence, ρ converges asymptotically towards zero in every state of the hybrid system, while guaranteeing that the tracked feature remains in view.

It is important to notice that the control law defined thus far may generate switches among different control laws with an increasing frequency as ρ decreases. To avoid this phenomenon (sometimes referred to as the *Zeno behavior* in the hybrid system literature), it is sufficient to introduce a sixth state in the system automaton, into which a transition from any other state is allowed when $\rho < \epsilon$, where ϵ represents a tolerable value of the residual position error. Once in this sixth state, the vehicle forward velocity u is set to zero, while the angular velocity is simply chosen as proportional to the error in the vehicle orientation, i.e. $\omega = k\gamma + \eta$. No exit condition is provided from this state. The practical stability of the proposed law is thus established to within a neighborhood $[\pm\epsilon, \pm\epsilon, \pm \arctan(\frac{\epsilon}{\sqrt{R^2-\epsilon^2}})]$ of the origin in the original coordinates (ξ_1, ξ_2, ξ_3).

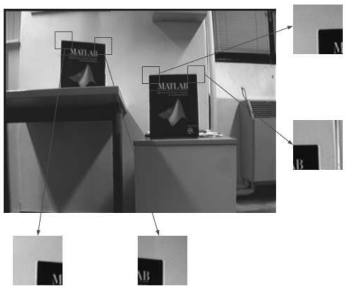

Fig. 4. Image grabbed from the target position. The four selected control features are represented in the small squares

Fig. 5. Left: Image grabbed from the starting position. Right: Image grabbed from the final position. It is possible to note the similarity between this image and the one on the previous figure.

4 Simulations and Experiments

In the simulation and experimental results presented in this section, the angle of view limit is fixed to $\Delta = \frac{\pi}{6}$. In the simulation reported in fig. 3, the unicycle achieves the goal notwithstanding the rather awkward initial position, with the selected feature situated between its start position and the desired one. Each cusp in the resulting trajectory corresponds to a switch among different states in the hybrid controller. Chattering near the equilibrium illustrates the Zeno behavior, which is circumvented by the practically stabilizing controller described above.

The experimental setup was comprised of a TRC LabMate vehicle, equipped with an analogue monochromatic camera Jai CVM-50 placed on the robot so that a vertical axis through the camera pinhole intersects the wheel axis in the midpoint. The controller is implemented on a common Linux based 300MHz PentiumII PC equipped with a Matrox MeteorI frame grabber. XVision (see [10]) is used to compute optical flow and to track features. The hardware communication between the robot and the PC is performed by a RS-232 serial cable. A few procedures have been used to improve the signal-to-noise ratio in grabbed images, as e.g. multiple

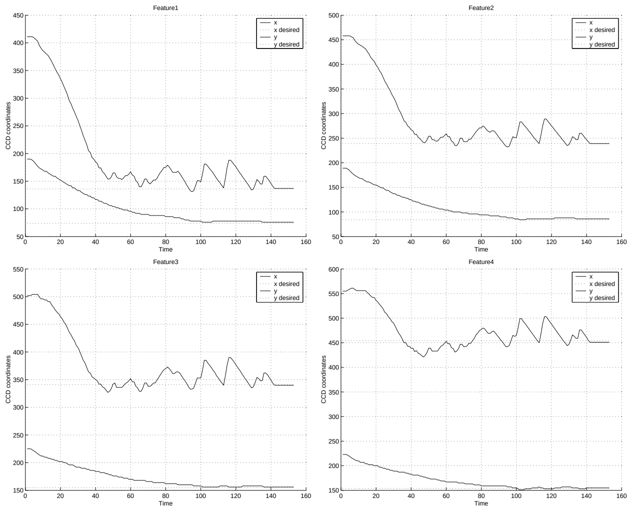

Fig. 6. Trajectory of the x and y coordinates in the image plane of the four features. The dotted lines represent the target values.

temporal windowing and filtering of grabbed images. To avoid the intrinsic analytical singularities on the inversion of the perspective projection, a semi-heuristic technique has been implemented, called *Feature Migration* that consists in mapping features (in both current and target images) away from the middle axis, to reject mismatches in the camera calibration and feature height parameters.

In the experiments, four features from the scene are used to implement the algorithm (see fig. 4). Although the theory above described has only been proved consistent for a single feature, we successfully applied a slightly modified version to the problem of keeping multiple features in view. The target image, recorded in a preliminary phase of the experiment, is reported in fig. 4. The image grabbed from the robot camera in the initial, offset configuration is in fig. 5, left, while the image grabbed at the end of the visually-servoed parking maneuver is shown on the right. Fig. 6 reports trajectories of the four selected features in the image plane, illustrating convergence as well as satisfaction of the field-of-view constraint.

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